

Galactic clusters with associated Cepheid variables – VII. Berkeley 58 and CG Cassiopeiae

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ABSTRACT

Photoelectric, photographic and CCD *UBV* photometry, spectroscopic observations and star counts are presented for the open cluster Berkeley 58 to examine a possible association with the 4.37 d Cepheid CG Cas. The cluster is difficult to separate from the early-type stars belonging to the Perseus spiral arm, in which it is located, but has reasonably well-defined parameters: an evolutionary age of $\sim 10^8$ yr, a mean reddening of $E(B - V) (B0) = 0.70 \pm 0.03$ s.e. and a distance of 3.03 ± 0.17 kpc ($V_0 - M_V = 12.40 \pm 0.12$ s.d.). CG Cas is a likely cluster coronal member on the basis of radial velocity, and its period increase of $+0.170 \pm 0.014$ s yr⁻¹ and large light amplitude describe a Cepheid in the third crossing of the instability strip lying slightly blueward of strip centre. Its inferred reddening and luminosity are $E(B - V) = 0.64 \pm 0.02$ s.e. and $\langle M_V \rangle = -3.06 \pm 0.12$. A possible K supergiant may also be a cluster member.

Key words: stars: evolution – Cepheids – open clusters and associations: individual: Berkeley 58.

1 INTRODUCTION

After the rediscovery in the early 1950s of spatial coincidences between Cepheids and open clusters by Irwin (1955, 1958), Eggen (see Sandage 1958) and Kholopov (1956), a number of searches for additional coincidences were made by Kraft (1957), van den Bergh (1957) and Tifft (1959), among others. Tifft's search resulted in the discovery of a near-spatial coincidence between the 4.37 d Cepheid CG Cassiopeiae and an anonymous open cluster, subsequently catalogued as Berkeley 58 (Setteducati & Weaver 1962), which lies

less than one cluster diameter to the west. The field is coincident with a portion of the Perseus spiral arm that is relatively rich in open clusters, and the cluster NGC 7790 with its three Cepheid members lies in close proximity. The possibility that CG Cas might be an outlying member of NGC 7790 was raised at one time by Efremov (1964a,b), and found some support in a star count analysis by Kovalenko (1968). More detailed star counts in the field (Turner 1985) indicate otherwise, as do the available proper motion data (Frolov 1974, 1977). The Cepheid does lie in the corona of Berkeley 58 (Turner 1985), although Frolov has argued that it is not a probable cluster member.

Given a probable distance of 3 kpc to both CG Cas and Berkeley 58 (e.g. Frolov 1979; Phelps & Janes 1994), it is not clear that existing proper motion data are precise enough to provide conclusive evidence pertaining to the cluster membership of CG Cas. The present study was therefore initiated in order to examine the case in more detail. As demonstrated here, there is strong evidence that CG Cas is a likely member of Berkeley 58 and that it can serve as a calibrator for the Cepheid period–luminosity (PL) relation.

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Table 1. Photoelectric *UBV* data for stars in Berkeley 58.

| Star | RA(2000) | Dec.(2000) | <i>V</i> | <i>B</i> − <i>V</i> | <i>U</i> − <i>B</i> | <i>n</i> | Notes |
|--------|-------------|------------|----------|---------------------|---------------------|----------|---------------------|
| CG Cas | 00 00 59.24 | 60 57 32.5 | 11.20 | 1.30 | +1.00 | 15 | F5-G1 I |
| 1 | 00 01 21.61 | 60 50 21.2 | 7.28 | 0.14 | −0.39 | 4 | |
| 2 | 00 00 47.68 | 60 48 49.8 | 9.80 | 0.27 | −0.10 | 4 | |
| 3 | 00 00 46.10 | 60 58 46.5 | 9.85 | 1.35 | +1.20 | 4 | |
| 4 | 00 00 42.77 | 61 03 26.1 | 10.02 | 0.58 | +0.02 | 4 | |
| 5 | 00 00 32.75 | 60 54 11.8 | 10.79 | 2.04 | +2.44 | 2 | (K II)? |
| 6 | 00 00 20.63 | 60 59 43.1 | 10.95 | 0.50 | −0.12 | 4 | B3-5 V ^a |
| 7 | 00 00 48.46 | 61 02 49.5 | 11.49 | 0.46 | −0.44 | 4 | B3-5 Vnn |
| 8 | 00 00 10.99 | 61 01 53.6 | 12.04 | 0.59 | +0.26 | 1 | |
| 9 | 23 59 45.42 | 60 56 28.1 | 12.11 | 0.57 | +0.48 | 3 | |
| 10 | 00 00 52.47 | 60 56 14.1 | 12.16 | 2.25 | +2.51 | 3 | |
| 11 | 00 00 40.07 | 61 03 21.9 | 12.35 | 0.50 | −0.25 | 4 | B2.5 V |
| 12 | 00 00 48.77 | 60 59 17.2 | 12.55 | 1.41 | +1.10 | 1 | |
| 13 | 00 00 33.91 | 60 57 58.7 | 12.78 | 0.62 | +0.09 | 4 | |
| 14 | 00 00 13.07 | 60 56 25.4 | 12.82 | 0.57 | +0.04 | 4 | |
| 15 | 00 00 25.25 | 61 00 29.8 | 13.12 | 1.59 | +1.50 | 3 | |
| 16 | 00 00 22.63 | 60 59 20.7 | 13.22 | 0.52 | +0.21 | 5 | |
| 17 | 00 00 25.83 | 60 57 58.2 | 13.30 | 0.83 | +0.55 | 4 | |
| 18 | 00 00 15.03 | 60 57 05.0 | 13.35 | 0.57 | +0.03 | 4 | B6 Vn |
| 19 | 00 00 09.46 | 60 57 47.6 | 13.36 | 0.72 | +0.44 | 2 | |
| 20 | 00 00 06.89 | 60 57 37.6 | 13.41 | 0.60 | +0.11 | 2 | |
| 21 | 00 00 36.95 | 61 02 55.7 | 13.41 | 0.66 | +0.49 | 3 | |
| 22 | 00 00 19.09 | 60 57 29.8 | 13.54 | 1.55 | +1.39 | 4 | |
| 23 | 00 00 16.44 | 60 57 08.8 | 13.60 | 0.56 | +0.00 | 4 | B5:: Vnn |
| 23 | 00 00 03.17 | 60 55 57.1 | 13.69 | 0.57 | +0.10 | 4 | B7 V |
| 25 | 00 00 56.70 | 61 01 12.0 | 13.71 | 1.43 | +1.23 | 2 | |
| 26 | 00 00 38.39 | 60 58 26.2 | 14.11 | 0.73 | +0.59 | 4 | |
| 27 | 00 00 57.31 | 60 58 54.9 | 14.14 | 0.84 | +0.26 | 4 | |
| 28 | 00 00 22.18 | 60 56 39.7 | 14.20 | 0.62 | +0.19 | 5 | |
| 29 | 00 00 15.73 | 60 56 08.6 | 14.70 | 0.57 | +0.18 | 4 | |
| 30 | 00 00 16.73 | 60 55 55.6 | 14.71 | 0.72 | +0.35 | 5 | double |
| .. | ... | ... | 15.46 | 0.76 | ... | CCD | |
| 31 | 00 00 10.33 | 60 56 25.4 | 14.74 | 1.57 | ... | 2 | |
| 32 | 00 00 19.10 | 60 57 44.5 | 14.75 | 0.62 | +0.44 | 3 | |
| 33 | 00 00 09.64 | 60 57 10.1 | 14.91 | 1.02 | +0.54 | 1 | |
| 34 | 00 00 11.54 | 60 55 19.5 | 15.06 | 1.09 | ... | 2 | |
| .. | ... | ... | 14.88 | 0.66 | +0.13 | CCD | |
| 35 | 00 00 18.88 | 60 56 24.1 | 15.09 | 0.61 | +0.25 | 4 | |
| 37 | 00 00 14.44 | 60 55 43.3 | 15.16 | 1.17 | ... | 1 | |
| 37 | 00 00 23.28 | 60 57 27.8 | 15.63 | 0.81 | +0.79 | 3 | |

^aV654 Cas (Berdnikov 1993).

2 OBSERVATIONAL DATA

A variety of observations were obtained for the present investigation. Table 1 presents photoelectric *UBV* photometry for bright members of Berkeley 58, obtained during observing runs at Kitt Peak National Observatory in 1981 September, 1982 August and 1984 August. The data, acquired using 1P21 photomultipliers and standard *UBV* filter sets used in conjunction with pulse-counting photometers on the No. 4 0.4-m, No. 2 0.9-m, and 1.3-m telescopes at Kitt Peak, have associated uncertainties typical of our previous investigations of Cepheid clusters (Turner 1992; Turner, Forbes & Pedreros 1992; Turner, Mandushev & Forbes 1994), namely standard internal errors for a single observation of ± 0.01 in *V* and *B* − *V*, and ± 0.02 in *U* − *B*, for stars brighter than *V* = 13. The estimated external errors for all but the faintest stars are similar in magnitude. The stars are identified by their numbering in Fig. 1, as well as by their 2000 coordinates in the Two Micron All-Sky Survey (2MASS) (Cutri et al. 2003); the number of individual observations for each star is given in Column 7 of Table 1.

Star 6 is the eclipsing system V654 Cas, for which Berdnikov (1993) cites photoelectric values of *V* and *B* − *V* outside of eclipse that are close to the values given here. Star 30 is a close optical double with components of nearly identical brightness. The photoelectric values apply to the combined light from both stars, whereas CCD observations provide uncontaminated data for the south-western star of the pair, as established by its CCD magnitude being 0.75 mag fainter. By contrast, the CCD *V* magnitude for star 35 is 0.21 mag brighter, which suggests possible variability in the object. Individual photoelectric observations for CG Cas are presented in Table 2.

Photographic *UBV* photometry was also obtained for stars in the nuclear and coronal regions of Berkeley 58 from photographic plates of the cluster field obtained in 1984 September using the 1.2-m Elginfield telescope of the University of Western Ontario. The star images were measured using the IRIS diaphragm photometer at Saint Mary's University, and were reduced to the *UBV* system and calibrated with reference to the photoelectric standards identified in Table 1 using the techniques discussed by Turner & Welch (1989).

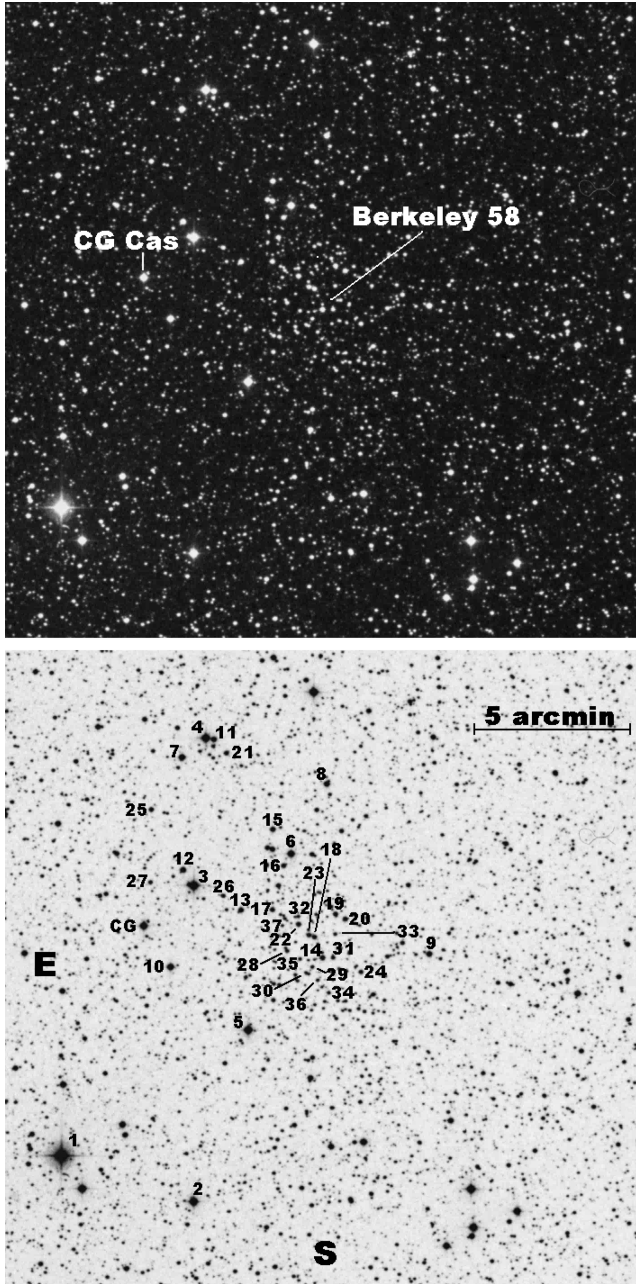


Figure 1. A finder chart for the field of Berkeley 58 from the red image of the Palomar Observatory Sky Survey. The field of view measures $20 \text{ arcmin} \times 20 \text{ arcmin}$ and is centred at 2000 coordinates: RA = $00^{\text{h}}00^{\text{m}}12^{\text{s}}.9$, Dec. = $+60^{\circ}56'07''$. The top image depicts the location of CG Cas relative to the cluster core, the lower image identifies photoelectrically observed stars. [The National Geographic Society-Palomar Observatory Sky Atlas (POSS-I) was made by the California Institute of Technology with grants from the National Geographic Society.]

The resulting data are presented in Table 3 in similar format to the data of Table 1, and the stars are identified by their 2000 coordinates. The photographic values for cluster stars in common with the CCD survey (Phelps & Janes 1994) agree very closely with the CCD values, when the latter are adjusted to the present system. However, earlier photographic *UBV* photometry of cluster stars by Frolov (1979) displays systematic differences relative to the present data. Since the present survey samples a much larger number of cluster

Table 2. Photoelectric *UBV* observations for CG Cas-siopeiae.

| HJD | V | B – V | U – B |
|---------------|-------|-------|-------|
| 244 4849.8938 | 11.37 | 1.28 | ... |
| 244 4854.8655 | 11.53 | 1.36 | ... |
| 244 4856.8539 | 11.22 | 1.14 | ... |
| 244 4857.8358 | 11.08 | 1.14 | ... |
| 244 4857.8689 | 11.09 | 1.16 | ... |
| 244 5197.9177 | 10.92 | 1.04 | 0.74 |
| 244 5205.9366 | 11.45 | 1.25 | 0.84 |
| 244 5206.8851 | 11.04 | 1.10 | 0.76 |
| 244 5933.8457 | 11.73 | 1.40 | 1.04 |
| 244 5935.8748 | 10.99 | 1.08 | 0.85 |
| 244 5937.8601 | 11.59 | 1.37 | 0.96 |
| 244 5938.8420 | 11.74 | 1.38 | 1.02 |
| 244 5939.8315 | 10.85 | 0.99 | 0.72 |
| 244 5941.8773 | 11.55 | 1.35 | 0.93 |
| 244 5942.7724 | 11.76 | 1.42 | 1.04 |

stars, no attempt was made to combine Frolov’s data with the present photometry.

CCD *UBV* photometry for stars in the nuclear region of Berkeley 58 was published previously by Phelps & Janes (1994), but for this study was recalibrated using the Table 1 stars as standards. The revised photometry for cluster stars is presented in Table 4, where the star numbers correspond to the scheme adopted by Phelps & Janes (1994), incremented by 1000. The stars are also identified by their 2000 coordinates. Since the *U*-band measurements have a much brighter limit than the *B* and *V* measures, the CCD photometry is less useful for studying the reddening in the field. But it is valuable for identifying the faint portion of the cluster main sequence.

Spectroscopic imaging of bright stars in Berkeley 58 was made in 1984 July and 1985 September using the Cassegrain spectrograph on the 1.8-m Plaskett telescope of the Dominion Astrophysical Observatory. The observations, at a dispersion of 15 \AA mm^{-1} and centred in the blue spectral region, were recorded photographically and later scanned for radial velocity measurement with the PDS microdensitometer at the David Dunlap Observatory of the University of Toronto (see Turner & Drilling 1984). It was also possible to estimate spectral types for the stars from the photographic spectra, with results presented in Table 1.

The field of the Cepheid CG Cas was also examined on archival images in the collections of Harvard College Observatory and Sternberg Astronomical Institute in order to obtain brightness estimates for the star and to construct seasonal light curves for comparison with a standard light curve constructed from photoelectric observations (Berdnikov 2007). The resulting data were used to estimate times of light maximum for the Cepheid and to track its O–C changes, the differences between observed (O) and computed (C) times of light maximum. Rate of period change, in conjunction with light amplitude, is an excellent diagnostic of the location of individual Cepheids in the instability strip (Turner, Abdel-Sabour Abdel-Latif & Berdnikov 2006a), such information providing an excellent parameter for comparison with what can be gleaned from information on the age of the surrounding stars provided by the cluster Hertzsprung–Russell diagram.

3 STAR COUNTS

The first step in studying Berkeley 58 involved star counts made using a photographic enlargement from a glass copy of the Palomar

Table 3. Photographic *UBV* Data for stars in Berkeley 58.

| Star | RA(2000) | Dec.(2000) | <i>V</i> | <i>B</i> − <i>V</i> | <i>U</i> − <i>B</i> | Star | RA(2000) | Dec.(2000) | <i>V</i> | <i>B</i> − <i>V</i> | <i>U</i> − <i>B</i> |
|------|-------------|-------------|----------|---------------------|---------------------|------|-------------|-------------|----------|---------------------|---------------------|
| 101 | 23 59 36.09 | +60 49 01.9 | 9.48 | 0.52 | −0.05 | 179 | 00 01 21.27 | +60 53 30.5 | 14.01 | 0.60 | +0.47 |
| 102 | 00 00 13.86 | +61 04 47.4 | 9.76 | 0.33 | +0.15 | 180 | 23 59 56.23 | +60 56 25.0 | 14.03 | 0.58 | +0.08 |
| 103 | 00 01 16.38 | +60 49 18.1 | 9.82 | 0.36 | −0.28 | 181 | 23 58 53.10 | +60 57 05.6 | 14.05 | 0.98 | +0.93 |
| 104 | 23 59 24.28 | +60 48 18.3 | 10.12 | 1.82 | +0.74 | 182 | 23 59 10.64 | +60 51 55.7 | 14.06 | 0.90 | +0.25 |
| 105 | 23 59 35.71 | +60 47 49.6 | 10.26 | 1.48 | +0.84 | 183 | 23 59 17.01 | +60 51 12.5 | 14.07 | 1.03 | +0.19 |
| 106 | 00 00 03.42 | +60 50 36.1 | 11.10 | 0.71 | −0.05 | 184 | 23 59 47.40 | +60 48 27.0 | 14.07 | 0.92 | +0.30 |
| 107 | 23 59 36.15 | +60 47 33.1 | 11.59 | 0.73 | +0.13 | 185 | 23 59 50.39 | +60 58 39.3 | 14.09 | 1.64 | ... |
| 108 | 23 59 42.73 | +60 47 16.1 | 11.60 | 2.12 | +2.67 | 186 | 00 00 49.32 | +60 48 06.5 | 14.10 | 0.91 | +0.08 |
| 109 | 23 59 07.11 | +60 55 06.8 | 11.67 | 1.51 | +1.09 | 187 | 00 00 24.36 | +60 55 35.8 | 14.11 | 0.63 | +0.09 |
| 110 | 00 01 05.72 | +60 51 17.7 | 11.83 | 1.10 | +0.67 | 188 | 23 59 49.10 | +60 50 32.7 | 14.13 | 1.07 | +0.35 |
| 111 | 00 00 11.91 | +60 50 28.6 | 11.85 | 0.53 | +0.17 | 189 | 23 58 57.51 | +60 58 24.8 | 14.14 | 1.31 | +0.85 |
| 112 | 00 01 20.84 | +60 52 34.9 | 11.92 | 1.31 | +0.95 | 190 | 23 59 22.80 | +60 57 04.3 | 14.14 | 0.52 | −0.12 |
| 113 | 00 01 21.00 | +61 00 48.4 | 12.08 | 0.47 | +0.31 | 191 | 23 59 05.78 | +61 01 43.6 | 14.15 | 0.79 | +0.70 |
| 114 | 00 01 10.16 | +61 03 50.0 | 12.17 | 0.88 | +0.34 | 192 | 23 59 46.99 | +61 03 27.0 | 14.16 | 0.58 | +0.38 |
| 115 | 00 01 20.63 | +60 55 32.1 | 12.37 | 0.56 | +0.12 | 193 | 00 00 55.95 | +61 03 02.4 | 14.16 | 0.55 | +0.19 |
| 116 | 00 00 07.17 | +60 48 48.4 | 12.39 | 0.80 | +0.52 | 194 | 00 00 04.13 | +60 51 51.7 | 14.17 | 0.86 | +0.54 |
| 117 | 23 58 53.91 | +60 56 37.2 | 12.40 | 0.78 | +0.52 | 195 | 00 00 37.31 | +60 46 36.1 | 14.17 | 1.09 | +0.43 |
| 118 | 23 59 10.07 | +60 55 48.6 | 12.41 | 0.78 | +0.52 | 196 | 23 59 37.23 | +61 01 47.2 | 14.19 | 0.71 | −0.04 |
| 119 | 23 59 40.81 | +60 51 12.4 | 12.41 | 1.35 | +1.05 | 197 | 00 01 14.97 | +60 54 01.9 | 14.20 | 0.87 | +0.26 |
| 120 | 00 00 24.15 | +61 05 54.4 | 12.51 | 1.65 | +1.73 | 198 | 23 59 07.54 | +60 59 40.0 | 14.21 | 0.80 | −0.01 |
| 121 | 00 00 16.78 | +60 52 39.3 | 12.54 | 0.71 | +0.22 | 199 | 23 59 49.51 | +60 59 23.1 | 14.25 | 0.97 | +0.85 |
| 122 | 00 01 05.00 | +60 50 58.3 | 12.58 | 0.30 | −0.02 | 200 | 00 00 26.46 | +60 50 28.7 | 14.30 | 0.66 | +0.52 |
| 123 | 23 59 11.61 | +61 02 04.7 | 12.59 | 1.64 | +0.71 | 201 | 00 00 51.81 | +60 46 54.6 | 14.30 | 0.86 | +0.22 |
| 124 | 23 59 43.07 | +61 03 17.7 | 12.63 | 0.52 | +0.20 | 202 | 00 00 23.33 | +60 51 42.1 | 14.31 | 0.81 | +0.21 |
| 125 | 23 59 06.63 | +60 53 17.9 | 12.78 | 0.60 | +0.33 | 203 | 23 59 34.32 | +60 59 24.9 | 14.32 | 0.97 | +0.31 |
| 126 | 00 01 05.84 | +60 59 50.1 | 12.82 | 0.47 | −0.01 | 204 | 23 59 17.67 | +60 54 45.5 | 14.34 | 1.06 | +0.33 |
| 127 | 00 01 33.08 | +60 53 08.0 | 12.90 | 0.53 | +0.41 | 205 | 00 00 11.83 | +61 05 55.5 | 14.34 | 0.97 | +0.41 |
| 128 | 00 00 57.98 | +61 04 02.5 | 12.97 | 0.56 | −0.20 | 206 | 23 59 41.42 | +60 51 28.7 | 14.40 | 0.58 | +0.40 |
| 129 | 00 00 25.44 | +60 59 52.4 | 13.05 | 1.66 | +1.57 | 207 | 00 00 25.66 | +60 50 43.7 | 14.43 | 0.76 | +0.18 |
| 130 | 23 59 03.83 | +60 51 31.8 | 13.08 | 0.97 | +0.23 | 208 | 23 59 42.89 | +61 02 29.1 | 14.46 | 0.54 | +0.03 |
| 131 | 23 58 54.24 | +60 54 15.1 | 13.11 | 1.46 | +1.48 | 209 | 00 00 58.82 | +60 55 01.6 | 14.47 | 0.96 | +0.59 |
| 132 | 00 00 26.72 | +60 59 55.4 | 13.12 | 0.69 | +0.29 | 210 | 00 00 00.99 | +61 00 51.4 | 14.48 | 0.97 | +0.78 |
| 133 | 23 59 19.28 | +60 50 11.7 | 13.20 | 0.56 | −0.04 | 211 | 00 01 25.15 | +61 02 11.0 | 14.51 | 0.79 | +0.56 |
| 134 | 23 59 24.57 | +60 55 27.9 | 13.30 | 0.65 | +0.40 | 212 | 23 59 23.62 | +60 59 41.4 | 14.52 | 0.77 | +0.36 |
| 135 | 23 59 59.65 | +61 04 09.7 | 13.32 | 1.32 | +1.21 | 213 | 00 00 26.84 | +60 49 40.7 | 14.55 | 1.00 | +0.51 |
| 136 | 00 00 27.04 | +60 46 43.8 | 13.32 | 1.10 | +0.22 | 214 | 00 01 21.95 | +61 02 29.5 | 14.55 | 0.94 | +0.62 |
| 137 | 00 00 28.97 | +60 47 58.6 | 13.39 | 0.89 | +0.24 | 215 | 00 00 25.83 | +60 55 38.0 | 14.57 | 0.65 | +0.28 |
| 138 | 00 00 21.24 | +60 51 10.6 | 13.40 | 0.49 | +0.20 | 216 | 23 59 06.22 | +60 53 57.6 | 14.58 | 1.10 | +0.46 |
| 139 | 00 01 16.05 | +61 02 45.5 | 13.44 | 0.83 | +0.55 | 217 | 00 01 18.21 | +61 01 23.9 | 14.59 | 1.08 | +0.49 |
| 140 | 23 59 41.35 | +61 05 46.4 | 13.45 | 0.69 | +0.25 | 218 | 00 00 16.19 | +60 53 17.5 | 14.59 | 2.38 | ... |
| 141 | 23 58 50.90 | +60 54 37.2 | 13.46 | 1.14 | +0.16 | 219 | 23 59 22.49 | +60 52 00.0 | 14.60 | 1.28 | +0.28 |
| 142 | 00 00 15.19 | +60 59 41.4 | 13.50 | 0.55 | +0.08 | 220 | 00 00 01.21 | +60 57 39.2 | 14.60 | 0.73 | +0.47 |
| 143 | 00 01 38.02 | +60 57 11.3 | 13.51 | 0.90 | +0.24 | 221 | 00 00 28.99 | +60 52 57.1 | 14.61 | 0.85 | +0.31 |
| 144 | 00 00 19.83 | +60 49 11.5 | 13.52 | 0.94 | +0.48 | 222 | 00 00 53.50 | +60 55 22.0 | 14.65 | 0.38 | +0.08 |
| 145 | 23 59 48.55 | +60 46 45.3 | 13.56 | 1.29 | +0.38 | 223 | 00 00 28.35 | +61 05 20.9 | 14.70 | 0.66 | −0.01 |
| 146 | 00 00 24.12 | +60 58 43.8 | 13.58 | 1.17 | +0.83 | 224 | 23 59 58.59 | +60 54 09.2 | 14.73 | 1.01 | +0.71 |
| 147 | 00 01 14.18 | +60 56 32.2 | 13.59 | 0.46 | +0.03 | 225 | 00 01 16.76 | +60 54 30.9 | 14.74 | 0.45 | −0.17 |
| 148 | 23 59 24.83 | +60 59 52.9 | 13.64 | 1.18 | ... | 226 | 00 00 18.03 | +60 51 36.5 | 14.75 | 0.76 | +0.37 |
| 149 | 00 00 07.41 | +61 00 24.1 | 13.64 | 0.61 | +0.23 | 227 | 23 59 32.73 | +60 59 21.3 | 14.79 | 0.79 | +0.48 |
| 150 | 00 00 01.78 | +60 48 57.5 | 13.68 | 0.61 | +0.30 | 228 | 00 00 10.51 | +60 58 01.8 | 14.79 | 0.77 | +0.43 |
| 151 | 23 58 55.65 | +60 52 55.3 | 13.70 | 0.75 | +0.27 | 229 | 23 59 40.12 | +60 57 06.2 | 14.81 | 0.96 | +0.33 |
| 152 | 23 59 09.72 | +60 55 33.4 | 13.71 | 0.93 | +0.31 | 230 | 00 00 20.17 | +60 55 54.3 | 14.86 | 0.68 | +0.28 |
| 153 | 23 59 52.24 | +60 56 50.2 | 13.72 | 0.56 | +0.13 | 231 | 00 00 02.48 | +60 54 42.4 | 14.88 | 0.45 | +0.20 |
| 154 | 00 00 49.48 | +61 03 45.2 | 13.72 | 1.44 | +0.78 | 232 | 23 59 49.71 | +60 53 31.9 | 14.90 | 0.76 | +0.39 |
| 155 | 00 01 38.95 | +60 57 40.8 | 13.73 | 0.53 | −0.01 | 233 | 00 00 04.01 | +60 54 47.8 | 14.93 | 0.61 | +0.19 |
| 156 | 00 00 27.09 | +61 05 48.8 | 13.74 | 1.06 | +0.32 | 234 | 00 00 33.02 | +60 55 51.9 | 14.96 | 1.05 | +0.44 |
| 157 | 23 59 34.31 | +60 49 44.5 | 13.75 | 1.14 | +0.47 | 235 | 00 00 09.83 | +60 54 30.1 | 15.00 | 0.69 | +0.21 |
| 158 | 00 00 44.24 | +60 46 52.3 | 13.75 | 0.65 | +0.08 | 236 | 00 00 44.08 | +61 05 04.7 | 15.00 | 0.48 | +0.49 |
| 159 | 00 00 31.53 | +60 46 32.0 | 13.76 | 0.84 | +0.34 | 237 | 00 00 09.76 | +61 05 34.5 | 15.12 | 0.38 | +0.26 |
| 160 | 00 00 38.87 | +60 53 10.0 | 13.77 | 0.53 | +0.09 | 238 | 23 59 42.48 | +60 56 18.8 | 15.14 | 0.59 | +0.36 |
| 161 | 23 59 26.17 | +60 49 45.8 | 13.78 | 1.45 | +1.09 | 239 | 00 00 04.71 | +60 57 45.6 | 15.15 | 0.67 | +0.44 |
| 162 | 23 59 41.19 | +61 04 51.7 | 13.79 | 0.54 | +0.27 | 240 | 23 59 19.79 | +61 00 25.5 | 15.16 | 0.70 | +0.13 |

Table 3 – *continued*

| Star | RA(2000) | Dec.(2000) | <i>V</i> | <i>B</i> – <i>V</i> | <i>U</i> – <i>B</i> | Star | RA(2000) | Dec.(2000) | <i>V</i> | <i>B</i> – <i>V</i> | <i>U</i> – <i>B</i> |
|------|-------------|-------------|----------|---------------------|---------------------|------|-------------|-------------|----------|---------------------|---------------------|
| 163 | 00 01 38.40 | +60 56 46.7 | 13.79 | 0.45 | +0.29 | 241 | 23 59 39.28 | +60 57 14.7 | 15.17 | 0.64 | +0.31 |
| 164 | 00 01 13.36 | +61 01 33.3 | 13.81 | 0.67 | +0.05 | 242 | 00 00 22.59 | +60 57 40.8 | 15.17 | 0.63 | +0.09 |
| 165 | 23 59 20.85 | +61 02 22.0 | 13.86 | 0.98 | +0.61 | 243 | 23 59 53.74 | +60 57 08.8 | 15.20 | 1.04 | +0.39 |
| 166 | 00 00 05.82 | +60 50 35.8 | 13.86 | 0.42 | +0.00 | 244 | 00 00 04.46 | +61 00 44.7 | 15.21 | 0.72 | +0.28 |
| 167 | 00 00 02.37 | +60 46 38.7 | 13.86 | 1.07 | +0.04 | 245 | 00 00 06.29 | +60 54 44.8 | 15.21 | 0.66 | +0.29 |
| 168 | 23 59 04.31 | +61 01 44.6 | 13.90 | 0.93 | +0.84 | 246 | 00 00 38.77 | +60 56 31.4 | 15.21 | 0.82 | +0.49 |
| 169 | 00 00 00.15 | +60 55 14.2 | 13.90 | 0.56 | +0.06 | 247 | 23 59 50.61 | +60 55 25.6 | 15.23 | 0.71 | ... |
| 170 | 23 59 03.70 | +61 01 50.1 | 13.91 | 0.88 | +0.87 | 248 | 23 59 57.24 | +60 55 02.8 | 15.24 | 0.69 | +0.29 |
| 171 | 00 01 09.90 | +60 52 54.7 | 13.93 | 0.94 | +0.43 | 249 | 23 59 55.11 | +60 53 44.8 | 15.26 | 0.71 | +0.31 |
| 172 | 00 01 08.86 | +60 58 34.1 | 13.94 | 1.04 | +0.66 | 250 | 00 00 08.27 | +60 56 42.6 | 15.35 | 0.94 | +0.61 |
| 173 | 23 59 27.78 | +60 55 30.8 | 13.97 | 0.67 | +0.49 | 251 | 00 00 14.46 | +60 57 47.6 | 15.42 | 0.77 | +0.39 |
| 174 | 00 00 13.79 | +61 01 04.7 | 13.98 | 0.98 | +0.48 | 252 | 00 00 15.30 | +60 54 57.0 | 15.42 | 0.72 | ... |
| 175 | 00 00 26.02 | +60 55 07.4 | 13.98 | 0.64 | +0.20 | 253 | 00 00 31.15 | +61 04 07.7 | 15.59 | 0.85 | ... |
| 176 | 00 01 10.45 | +60 58 30.7 | 13.99 | 0.68 | +0.42 | 254 | 00 00 10.69 | +60 55 58.4 | 15.62 | 0.79 | ... |
| 177 | 00 00 11.44 | +60 51 41.8 | 14.01 | 0.56 | +0.12 | 255 | 00 00 06.19 | +60 56 32.6 | 15.79 | 0.56 | +0.23 |
| 178 | 00 01 16.45 | +60 58 25.5 | 14.01 | 1.05 | +0.21 | | | | | | |

Observatory Sky Survey (POSS) E plate for the field. Strip counts in several different orientations delineated the cluster centre, followed by ring counts illustrated in Fig. 2; the centre of symmetry is located at RA = 00^h00^m12^s.9, Dec. = +60°56′07″ (2000). The upper portion of Fig. 2 illustrates ring counts for stars detected on the 2MASS survey (Cutri et al. 2003) to the survey limit, whereas the lower portion shows star counts from the POSS-E plate to two different magnitude limits.

The counts from the 2MASS survey were made without regard for overlap with the star cluster NGC 7790, which lies 23 arcmin to the north-west of Berkeley 58, whereas the counts from the POSS-E plate were restricted beyond 11 arcmin from the cluster centre to sectors that avoided overlap with the outlying regions of NGC 7790. The effect of contamination from the coronal region of NGC 7790 is detectable in the 2MASS star counts beyond roughly 12 arcmin from the cluster centre, but because of restrictions imposed by the location of Berkeley 58 on the POSS, we were unable to establish uncontaminated star counts from the POSS-E plate beyond about 15 arcmin from the cluster centre. Nevertheless, the two sets of counts appear to yield similar parameters for the inner regions of the cluster. Berkeley 58 is estimated to have a nuclear radius of $r_n \simeq 4.5$ arcmin (4.0 pc) in the notation of Kholopov (1969), whereas the coronal (or tidal) radius is estimated to be $R_c \simeq 11$ arcmin (9.7 pc) from the trends in the 2MASS star densities as well as the apparent flattening of the POSS-E star densities in the outermost rings.

Star counts predict a total of 197 ± 27 members brighter than the limit of the 2MASS survey lying within 5 arcmin of the cluster centre, 487 ± 82 members within 11 arcmin of the cluster centre, field stars within the same regions being 715 and 4835, respectively. Field stars clearly outnumber cluster members in both regions. CG Cas is located 5.8 arcmin from the centre of Berkeley 58, in the cluster coronal region just beyond its nuclear boundaries. Although not projected on the core of Berkeley 58, CG Cas is spatially coincident with the cluster, which occupies most of the field of Fig. 1.

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Fig. 3 is a *UBV* colour–colour diagram for the field of Berkeley 58 surveyed in this study, as constructed from the data of Tables 1, 3 and 4. The phase-averaged data for CG Cas are from Berdnikov (2007). A reddened sequence of B- and A-type cluster members

can be detected in the data, but a cluster reddening of $E(B - V) \simeq 0.7$ places them in a section of the colour–colour diagram where they can be confused photometrically with unreddened, foreground, G-type stars. For that reason, it becomes essential to make the process of photometric identification of likely spectral classes for individual stars as reliable as possible, through the use of a well-established interstellar extinction relation. The spectral types obtained for six of the B-type, photoelectrically observed, cluster stars imply a reddening law for Berkeley 58 described by $E(U - B)/E(B - V) = 0.75$, along with a small curvature term (Turner 1989), identical to the reddening slope found previously for star clusters spatially adjacent to Berkeley 58 (Turner 1976b). Berkeley 58 stars were therefore dereddened with such a relationship, except for late-type stars where a steeper relationship was adopted, dependent upon the likely intrinsic colours of the stars.

Fig. 3 data indicate an absence of any unreddened O-, B- or A-type stars in the observed sample. That feature is confirmed by available 2MASS data for the observed stars (Cutri et al. 2003), which are depicted in the *JHK* colour–colour diagram of Fig. 4. An intrinsic relation for main-sequence stars in the 2MASS system was constructed from 2MASS observations of unreddened standard stars and stars in open clusters of uniform reddening (e.g. Turner 1996b), adjusted with a reddening slope $E(H - K)/E(J - H) = 0.55$, as derived from reddened stars of known spectral type. The number of cluster stars with *U*-band observations is a small fraction of the total sample, so Fig. 4 contains many more stars than Fig. 3. The selection of 2MASS data was also not restricted according to the magnitude of cited uncertainties in the data, so several points in Fig. 4 display unusually large scatter. It seems clear, however, that the sample of cluster stars surveyed consists mainly of stars reddened by $E(J - H) \geq 0.1$, which corresponds to $E(B - V) \geq 0.36$.

The correlation of reddening with distance towards Berkeley 58 was established from the available *UBV* photometry by dereddening the colours for individual stars in conjunction with a copy of the POSS field on which derived colour excesses $E(B - V)$ were recorded as they were obtained, with multiple solutions resolved by reference to the reddenings for spatially adjacent stars as well as by the reddenings derived for the stars from their 2MASS colours (Fig. 4). In most cases, the smaller *JHK* reddening of stars relative to those obtained from *UBV* colours was sufficient to resolve

Table 4. CCD *UBV* data for stars in the nucleus of Berkeley 58.

| Star | RA(2000) | Dec.(2000) | <i>V</i> | <i>B</i> − <i>V</i> | <i>U</i> − <i>B</i> | Star | RA(2000) | Dec.(2000) | <i>V</i> | <i>B</i> − <i>V</i> | <i>U</i> − <i>B</i> |
|------|-------------|-------------|----------|---------------------|---------------------|------|-------------|-------------|----------|---------------------|---------------------|
| 1001 | 23 59 13.34 | +60 54 41.7 | 16.66 | 0.95 | ... | 1192 | 23 59 59.79 | +60 53 21.0 | 16.89 | 1.08 | ... |
| 1002 | 23 59 13.58 | +60 55 25.5 | 16.14 | 0.73 | ... | 1193 | 00 00 00.31 | +60 54 14.4 | 16.54 | 1.36 | ... |
| 1003 | 23 59 12.82 | +60 57 28.6 | 15.57 | 1.08 | ... | 1194 | 00 00 04.01 | +61 01 58.9 | 16.48 | 1.09 | ... |
| 1004 | 23 59 12.99 | +60 56 53.1 | 15.94 | 0.72 | ... | 1195 | 00 00 01.08 | +60 55 09.3 | 16.33 | 1.02 | ... |
| 1005 | 23 59 12.00 | +60 53 31.9 | 17.75 | 1.04 | ... | 1196 | 00 00 02.86 | +60 58 36.1 | 15.55 | 0.70 | ... |
| 1006 | 23 59 14.92 | +61 00 43.2 | 15.93 | 1.05 | ... | 1197 | 00 00 04.28 | +61 01 21.8 | 17.45 | 0.68 | ... |
| 1007 | 23 59 15.93 | +61 02 43.6 | 17.47 | 1.05 | ... | 1199 | 00 00 04.99 | +61 02 05.1 | 17.21 | 1.35 | ... |
| 1008 | 23 59 16.20 | +60 53 18.5 | 17.02 | 0.95 | ... | 1200 | 00 00 03.25 | +60 57 25.0 | 16.32 | 1.07 | ... |
| 1009 | 23 59 15.49 | +61 00 26.4 | 16.92 | 1.26 | ... | 1201 | 00 00 01.48 | +60 52 41.4 | 16.07 | 1.62 | ... |
| 1010 | 23 59 16.63 | +61 01 54.6 | 17.06 | 1.13 | ... | 1203 | 00 00 01.53 | +60 52 25.7 | 15.75 | 0.87 | ... |
| 1011 | 23 59 14.72 | +60 56 52.5 | 17.15 | 1.16 | ... | 1205 | 00 00 05.80 | +61 01 13.8 | 16.77 | 0.96 | ... |
| 1012 | 23 59 14.95 | +60 57 04.1 | 16.56 | 1.09 | ... | 1206 | 00 00 02.37 | +60 53 20.1 | 16.78 | 0.91 | ... |
| 1013 | 23 59 16.05 | +60 59 17.2 | 14.57 | 1.71 | ... | 1207 | 00 00 02.97 | +60 54 13.8 | 15.84 | 0.93 | ... |
| 1014 | 23 59 13.40 | +60 52 36.2 | 16.16 | 0.85 | ... | 1208 | 00 00 02.45 | +60 52 53.0 | 16.66 | 1.11 | ... |
| 1015 | 23 59 14.93 | +60 56 09.3 | 16.38 | 1.08 | ... | 1209 | 00 00 04.49 | +60 57 29.5 | 16.99 | 1.00 | ... |
| 1016 | 23 59 13.64 | +60 53 08.8 | 16.38 | 1.00 | ... | 1211 | 00 00 05.98 | +60 59 42.6 | 17.07 | 0.88 | ... |
| 1017 | 23 59 15.15 | +60 56 21.8 | 16.61 | 0.77 | ... | 1213 | 00 00 04.22 | +60 54 26.9 | 16.61 | 0.64 | ... |
| 1018 | 23 59 15.09 | +60 56 05.4 | 16.29 | 0.99 | ... | 1214 | 00 00 04.64 | +60 55 16.8 | 16.97 | 0.91 | ... |
| 1019 | 23 59 15.91 | +60 57 50.1 | 18.08 | 0.92 | ... | 1215 | 00 00 03.56 | +60 52 40.1 | 15.33 | 1.00 | ... |
| 1020 | 23 59 17.35 | +61 00 48.8 | 17.05 | 1.04 | ... | 1216 | 00 00 08.29 | +61 02 39.0 | 15.44 | 0.86 | ... |
| 1021 | 23 59 16.51 | +60 58 48.2 | 17.44 | 0.96 | ... | 1218 | 00 00 05.99 | +60 56 57.7 | 17.20 | 1.00 | ... |
| 1022 | 23 59 16.58 | +60 58 23.4 | 16.62 | 1.29 | ... | 1220 | 00 00 07.02 | +60 58 12.1 | 16.78 | 0.81 | ... |
| 1023 | 23 59 17.72 | +61 00 12.3 | 17.82 | 0.94 | ... | 1222 | 00 00 05.08 | +60 53 18.6 | 16.86 | 1.08 | ... |
| 1024 | 23 59 15.99 | +60 55 24.2 | 17.13 | 0.86 | ... | 1223 | 00 00 05.00 | +60 53 09.0 | 17.75 | 1.36 | ... |
| 1025 | 23 59 17.49 | +60 58 26.9 | 15.97 | 0.96 | ... | 1224 | 00 00 07.16 | +60 56 52.8 | 17.97 | 1.16 | ... |
| 1026 | 23 59 17.43 | +60 57 55.6 | 15.08 | 1.07 | ... | 1225 | 00 00 05.25 | +60 52 21.2 | 17.19 | 1.34 | ... |
| 1027 | 23 59 16.20 | +60 53 18.5 | 16.77 | 0.98 | ... | 1227 | 00 00 06.82 | +60 55 59.0 | 16.19 | 0.75 | ... |
| 1028 | 23 59 17.45 | +60 55 32.8 | 16.04 | 0.98 | ... | 1228 | 00 00 08.19 | +60 58 11.0 | 15.91 | 0.90 | ... |
| 1029 | 23 59 19.01 | +60 59 14.0 | 17.23 | 0.98 | ... | 1229 | 00 00 06.17 | +60 53 07.6 | 17.72 | 1.63 | ... |
| 1030 | 23 59 17.76 | +60 55 55.7 | 15.77 | 0.66 | ... | 1230 | 00 00 06.64 | +60 53 58.4 | 18.39 | 1.09 | ... |
| 1031 | 23 59 17.95 | +60 56 16.5 | 16.04 | 0.93 | ... | 1231 | 00 00 08.71 | +60 58 24.0 | 16.19 | 1.15 | ... |
| 1034 | 23 59 20.75 | +61 01 52.0 | 17.54 | 1.79 | ... | 1232 | 00 00 07.31 | +60 55 02.8 | 15.75 | 1.66 | ... |
| 1035 | 23 59 20.78 | +61 02 08.0 | 15.90 | 1.54 | ... | 1233 | 00 00 09.48 | +60 59 59.1 | 16.64 | 1.46 | ... |
| 1036 | 23 59 19.06 | +60 57 40.1 | 16.92 | 1.36 | ... | 1234 | 00 00 09.94 | +61 01 12.3 | 17.67 | 1.03 | ... |
| 1037 | 23 59 21.76 | +61 02 13.7 | 16.38 | 0.98 | ... | 1235 | 00 00 10.21 | +61 01 33.5 | 15.35 | 1.51 | ... |
| 1038 | 23 59 21.29 | +61 00 29.4 | 17.49 | 1.36 | ... | 1236 | 00 00 07.39 | +60 54 47.3 | 16.33 | 0.78 | ... |
| 1039 | 23 59 20.03 | +60 57 05.3 | 17.09 | 1.10 | ... | 1238 | 00 00 11.06 | +61 02 06.7 | 17.04 | 1.15 | ... |
| 1040 | 23 59 19.65 | +60 54 46.4 | 16.78 | 1.37 | ... | 1240 | 00 00 08.12 | +60 54 44.7 | 16.01 | 0.70 | ... |
| 1041 | 23 59 18.71 | +60 52 29.0 | 16.54 | 1.71 | ... | 1242 | 00 00 07.23 | +60 52 10.7 | 16.73 | 2.04 | ... |
| 1042 | 23 59 21.26 | +60 58 02.7 | 17.11 | 1.06 | ... | 1243 | 00 00 08.18 | +60 54 17.3 | 16.51 | 0.85 | ... |
| 1043 | 23 59 19.98 | +60 54 26.0 | 17.52 | 1.04 | ... | 1244 | 00 00 12.27 | +61 03 25.6 | 16.87 | 1.17 | ... |
| 1044 | 23 59 21.95 | +60 55 40.3 | 16.45 | 1.07 | ... | 1246 | 00 00 11.22 | +61 00 50.2 | 17.57 | 1.13 | ... |
| 1047 | 23 59 21.29 | +61 00 29.4 | 17.71 | 1.02 | ... | 1248 | 00 00 09.33 | +60 55 04.1 | 15.40 | 0.99 | ... |
| 1048 | 23 59 25.23 | +61 01 44.0 | 18.17 | 0.76 | ... | 1249 | 00 00 07.98 | +60 51 49.3 | 17.45 | 1.01 | ... |
| 1050 | 23 59 25.80 | +61 00 40.3 | 16.88 | 0.78 | ... | 1251 | 00 00 11.50 | +60 58 32.1 | 16.49 | 1.22 | ... |
| 1051 | 23 59 25.17 | +60 57 56.3 | 16.88 | 0.90 | ... | 1254 | 00 00 10.42 | +60 54 35.7 | 16.55 | 1.19 | ... |
| 1052 | 23 59 23.33 | +60 53 29.9 | 18.07 | 1.16 | ... | 1255 | 00 00 13.91 | +61 02 16.6 | 16.71 | 1.12 | ... |
| 1053 | 23 59 24.72 | +60 56 16.0 | 15.59 | 0.68 | ... | 1256 | 00 00 12.77 | +60 59 23.5 | 17.12 | 1.54 | ... |
| 1054 | 23 59 25.52 | +60 58 09.7 | 16.68 | 0.99 | ... | 1258 | 00 00 13.64 | +61 00 37.4 | 16.37 | 0.74 | ... |
| 1056 | 23 59 28.15 | +61 02 06.4 | 15.35 | 1.05 | ... | 1260 | 00 00 15.23 | +61 03 28.2 | 16.50 | 1.06 | ... |
| 1057 | 23 59 28.38 | +61 02 40.0 | 17.79 | 1.26 | ... | 1261 | 00 00 11.36 | +60 53 43.7 | 16.56 | 1.19 | ... |
| 1058 | 23 59 25.80 | +60 56 12.5 | 16.11 | 0.74 | ... | 1262 | 00 00 15.48 | +61 02 13.4 | 17.22 | 1.27 | ... |
| 1059 | 23 59 27.56 | +61 00 01.0 | 17.21 | 1.16 | ... | 1263 | 00 00 13.05 | +60 56 35.4 | 16.28 | 1.10 | ... |
| 1060 | 23 59 28.13 | +61 01 08.2 | 18.27 | 1.23 | ... | 1267 | 00 00 13.09 | +60 54 50.0 | 16.30 | 0.79 | ... |
| 1061 | 23 59 27.05 | +60 57 08.2 | 16.71 | 1.09 | ... | 1268 | 00 00 13.36 | +60 55 30.2 | 16.32 | 0.81 | ... |
| 1062 | 23 59 27.87 | +60 58 36.6 | 16.25 | 0.79 | ... | 1270 | 00 00 14.81 | +60 58 25.1 | 15.14 | 0.83 | +0.39 |
| 1063 | 23 59 27.43 | +60 56 19.0 | 16.13 | 0.81 | ... | 1271 | 00 00 12.88 | +60 53 46.9 | 16.18 | 0.99 | ... |
| 1064 | 23 59 28.87 | +60 59 01.0 | 17.09 | 1.58 | ... | 1272 | 00 00 14.21 | +60 56 52.2 | 16.68 | 0.84 | ... |
| 1066 | 23 59 31.06 | +61 03 11.5 | 17.93 | 0.73 | ... | 1273 | 00 00 16.48 | +61 01 21.1 | 16.21 | 1.21 | ... |
| 1067 | 23 59 29.96 | +61 00 22.6 | 16.55 | 0.79 | ... | 1276 | 00 00 15.16 | +60 56 22.6 | 16.45 | 0.90 | ... |
| 1068 | 23 59 32.09 | +61 03 09.1 | 15.75 | 0.71 | ... | 1277 | 00 00 18.04 | +61 02 55.1 | 15.96 | 1.18 | ... |
| 1069 | 23 59 28.50 | +60 53 37.0 | 16.50 | 1.90 | ... | 1278 | 00 00 15.74 | +60 57 35.8 | 15.97 | 0.73 | ... |
| 1070 | 23 59 30.44 | +60 57 43.2 | 16.01 | 0.80 | ... | 1280 | 00 00 15.73 | +60 56 08.6 | 14.63 | 0.62 | ... |

Table 4 – *continued*

| Star | RA(2000) | Dec.(2000) | V | B – V | U – B | Star | RA(2000) | Dec.(2000) | V | B – V | U – B |
|------|-------------|-------------|-------|-------|-------|------|-------------|-------------|-------|-------|-------|
| 1071 | 23 59 28.90 | +60 53 12.9 | 15.76 | 0.77 | ... | 1281 | 00 00 19.00 | +61 03 27.4 | 16.65 | 1.19 | ... |
| 1072 | 23 59 33.99 | +61 03 29.5 | 16.00 | 1.24 | ... | 1282 | 00 00 16.52 | +60 57 18.9 | 15.61 | 0.66 | ... |
| 1073 | 23 59 34.17 | +61 03 37.8 | 16.26 | 1.10 | ... | 1284 | 00 00 17.90 | +61 00 03.0 | 17.22 | 1.01 | ... |
| 1074 | 23 59 29.68 | +60 52 38.3 | 15.41 | 0.84 | ... | 1285 | 00 00 16.72 | +60 56 21.5 | 16.29 | 0.79 | ... |
| 1075 | 23 59 30.93 | +60 54 47.6 | 14.80 | 0.79 | ... | 1286 | 00 00 16.19 | +60 54 46.9 | 16.00 | 0.80 | ... |
| 1076 | 23 59 32.61 | +60 57 52.0 | 17.72 | 1.53 | ... | 1288 | 00 00 18.80 | +61 00 28.6 | 16.45 | 1.10 | ... |
| 1077 | 23 59 33.73 | +60 59 41.7 | 16.10 | 1.17 | ... | 1289 | 00 00 15.18 | +60 52 03.5 | 15.89 | 0.69 | ... |
| 1078 | 23 59 34.68 | +61 01 48.7 | 15.65 | 0.98 | ... | 1291 | 00 00 17.51 | +60 54 33.5 | 16.94 | 0.93 | ... |
| 1079 | 23 59 31.86 | +60 54 42.7 | 17.01 | 1.04 | ... | 1292 | 00 00 19.32 | +60 58 39.6 | 17.47 | 1.00 | ... |
| 1080 | 23 59 31.86 | +60 54 42.7 | 16.97 | 1.49 | ... | 1296 | 00 00 17.29 | +60 53 12.4 | 15.17 | 0.63 | +0.35 |
| 1081 | 23 59 32.85 | +60 56 59.5 | 16.14 | 1.15 | ... | 1297 | 00 00 18.05 | +60 54 52.8 | 18.48 | 0.75 | ... |
| 1082 | 23 59 34.12 | +60 59 35.0 | 17.33 | 1.21 | ... | 1300 | 00 00 17.98 | +60 53 30.9 | 16.95 | 0.94 | ... |
| 1083 | 23 59 34.86 | +61 00 37.8 | 18.19 | 1.32 | ... | 1301 | 00 00 19.95 | +60 57 43.7 | 16.49 | 0.83 | ... |
| 1084 | 23 59 35.83 | +61 02 46.5 | 17.06 | 1.23 | ... | 1302 | 00 00 22.49 | +61 03 13.7 | 17.65 | 1.30 | ... |
| 1085 | 23 59 33.94 | +60 58 15.1 | 15.44 | 1.02 | ... | 1303 | 00 00 19.95 | +60 56 56.2 | 16.21 | 0.76 | ... |
| 1086 | 23 59 32.21 | +60 54 02.5 | 17.03 | 1.15 | ... | 1304 | 00 00 19.64 | +60 55 42.7 | 15.41 | 0.82 | +0.40 |
| 1087 | 23 59 37.51 | +61 03 44.1 | 17.09 | 1.08 | ... | 1305 | 00 00 19.28 | +60 54 20.6 | 17.33 | 0.88 | ... |
| 1089 | 23 59 36.54 | +61 00 02.4 | 16.53 | 0.98 | ... | 1306 | 00 00 22.71 | +61 01 56.3 | 15.56 | 0.97 | ... |
| 1090 | 23 59 36.12 | +60 58 55.6 | 17.09 | 0.96 | ... | 1308 | 00 00 20.21 | +60 55 46.7 | 16.06 | 0.82 | ... |
| 1091 | 23 59 34.93 | +60 55 57.4 | 16.78 | 0.86 | ... | 1309 | 00 00 22.89 | +61 01 31.3 | 15.37 | 0.69 | ... |
| 1092 | 23 59 37.51 | +61 01 02.7 | 17.19 | 1.11 | ... | 1310 | 00 00 19.89 | +60 54 14.5 | 17.37 | 0.94 | ... |
| 1093 | 23 59 34.11 | +60 52 36.4 | 17.45 | 1.28 | ... | 1312 | 00 00 23.20 | +61 00 33.3 | 16.07 | 0.97 | ... |
| 1094 | 23 59 35.83 | +60 56 22.0 | 15.86 | 0.73 | ... | 1313 | 00 00 19.55 | +60 52 05.6 | 17.12 | 1.32 | ... |
| 1095 | 23 59 36.21 | +60 56 32.0 | 17.57 | 1.22 | ... | 1316 | 00 00 22.99 | +60 58 18.8 | 16.89 | 0.91 | ... |
| 1096 | 23 59 37.50 | +60 59 19.8 | 16.08 | 1.68 | ... | 1317 | 00 00 22.86 | +60 57 46.9 | 16.83 | 1.07 | ... |
| 1097 | 23 59 35.15 | +60 52 52.6 | 15.92 | 1.19 | ... | 1318 | 00 00 22.56 | +60 56 45.6 | 15.96 | 0.95 | ... |
| 1098 | 23 59 36.03 | +60 54 59.3 | 16.84 | 1.25 | ... | 1319 | 00 00 22.88 | +60 57 13.2 | 18.10 | 0.97 | ... |
| 1099 | 23 59 39.28 | +61 02 43.6 | 16.79 | 0.52 | ... | 1321 | 00 00 25.60 | +61 02 35.0 | 17.07 | 0.99 | ... |
| 1100 | 23 59 39.64 | +61 02 33.1 | 17.46 | 0.83 | ... | 1322 | 00 00 23.71 | +60 57 48.7 | 15.95 | 1.13 | ... |
| 1101 | 23 59 37.38 | +60 55 12.4 | 17.15 | 1.26 | ... | 1325 | 00 00 22.62 | +60 53 54.3 | 16.47 | 0.84 | ... |
| 1102 | 23 59 38.35 | +60 55 38.7 | 16.63 | 0.75 | ... | 1327 | 00 00 23.38 | +60 55 03.2 | 16.46 | 0.85 | ... |
| 1103 | 23 59 40.10 | +60 59 35.4 | 15.86 | 1.60 | ... | 1329 | 00 00 24.44 | +60 55 48.5 | 17.27 | 1.82 | ... |
| 1104 | 23 59 38.95 | +60 56 32.1 | 17.02 | 1.16 | ... | 1330 | 00 00 25.71 | +60 58 11.2 | 16.87 | 0.95 | ... |
| 1106 | 23 59 41.29 | +61 00 12.6 | 15.25 | 0.96 | ... | 1331 | 00 00 27.70 | +61 02 34.9 | 16.66 | 1.06 | ... |
| 1108 | 23 59 38.51 | +60 52 55.3 | 16.57 | 0.77 | ... | 1334 | 00 00 25.39 | +60 56 42.7 | 16.73 | 1.13 | ... |
| 1110 | 23 59 40.09 | +60 55 57.7 | 17.86 | 1.22 | ... | 1335 | 00 00 26.62 | +60 58 55.3 | 16.69 | 1.00 | ... |
| 1112 | 23 59 43.52 | +61 03 38.3 | 16.36 | 1.10 | ... | 1336 | 00 00 28.63 | +61 02 50.2 | 17.01 | 1.02 | ... |
| 1113 | 23 59 43.33 | +61 02 52.0 | 16.41 | 0.89 | ... | 1337 | 00 00 25.72 | +60 56 07.5 | 16.60 | 0.85 | ... |
| 1114 | 23 59 39.50 | +60 53 42.6 | 16.37 | 0.97 | ... | 1338 | 00 00 24.86 | +60 54 02.0 | 16.25 | 1.18 | ... |
| 1115 | 23 59 40.32 | +60 55 03.8 | 15.39 | 0.79 | +0.25 | 1340 | 00 00 27.13 | +60 58 13.9 | 18.06 | 1.69 | ... |
| 1116 | 23 59 40.65 | +60 54 36.6 | 16.49 | 0.82 | ... | 1341 | 00 00 26.64 | +60 56 47.9 | 16.46 | 1.01 | ... |
| 1117 | 23 59 41.73 | +60 56 35.4 | 15.32 | 0.86 | ... | 1343 | 00 00 26.18 | +60 54 16.6 | 16.25 | 1.04 | ... |
| 1119 | 23 59 43.50 | +60 58 13.5 | 17.14 | 1.22 | ... | 1344 | 00 00 29.86 | +61 01 29.8 | 16.83 | 1.34 | ... |
| 1120 | 23 59 45.69 | +61 02 26.1 | 15.32 | 1.07 | ... | 1345 | 00 00 25.91 | +60 52 38.3 | 17.19 | 0.86 | ... |
| 1121 | 23 59 42.53 | +60 53 55.6 | 16.28 | 0.72 | ... | 1346 | 00 00 29.97 | +61 01 08.2 | 17.60 | 0.84 | ... |
| 1122 | 23 59 46.20 | +61 02 01.7 | 17.52 | 1.25 | ... | 1347 | 00 00 27.25 | +60 54 15.9 | 16.34 | 0.97 | ... |
| 1123 | 23 59 43.41 | +60 55 23.6 | 17.04 | 0.84 | ... | 1348 | 00 00 28.62 | +60 56 47.1 | 16.96 | 1.08 | ... |
| 1125 | 23 59 44.33 | +60 56 58.6 | 16.72 | 0.89 | ... | 1349 | 00 00 28.30 | +60 55 49.0 | 15.64 | 0.72 | ... |
| 1126 | 23 59 43.56 | +60 55 04.0 | 16.21 | 2.03 | ... | 1350 | 00 00 27.82 | +60 53 48.3 | 16.70 | 1.70 | ... |
| 1127 | 23 59 47.02 | +61 03 03.1 | 17.31 | 0.91 | ... | 1351 | 00 00 27.71 | +60 53 14.2 | 15.58 | 0.80 | +0.36 |
| 1128 | 23 59 46.62 | +61 01 32.7 | 15.44 | 0.84 | ... | 1352 | 00 00 31.85 | +61 02 28.5 | 17.52 | 1.11 | ... |
| 1129 | 23 59 46.50 | +60 59 05.0 | 16.56 | 1.29 | ... | 1353 | 00 00 31.05 | +61 00 30.3 | 16.71 | 0.95 | ... |
| 1131 | 23 59 45.21 | +60 55 29.1 | 16.97 | 0.88 | ... | 1354 | 00 00 32.37 | +61 03 24.7 | 17.55 | 1.31 | ... |
| 1132 | 23 59 48.63 | +61 02 17.7 | 16.29 | 1.05 | ... | 1355 | 00 00 28.37 | +60 53 26.7 | 18.03 | 1.30 | ... |
| 1133 | 23 59 45.25 | +60 53 56.4 | 17.55 | 1.09 | ... | 1356 | 00 00 31.96 | +61 00 51.7 | 16.23 | 0.84 | ... |
| 1134 | 23 59 45.19 | +60 53 10.1 | 16.73 | 1.04 | ... | 1357 | 00 00 29.40 | +60 54 53.3 | 17.67 | 1.61 | ... |
| 1135 | 23 59 46.91 | +60 57 15.7 | 16.02 | 0.86 | ... | 1358 | 00 00 33.10 | +61 02 31.4 | 15.83 | 1.57 | ... |
| 1136 | 23 59 49.94 | +61 02 53.1 | 17.51 | 1.62 | ... | 1360 | 00 00 31.91 | +60 55 55.0 | 16.47 | 1.13 | ... |
| 1137 | 23 59 46.12 | +60 53 22.2 | 16.08 | 0.95 | ... | 1361 | 00 00 35.02 | +61 02 12.6 | 17.79 | 0.75 | ... |
| 1138 | 23 59 49.74 | +61 00 38.8 | 15.68 | 0.70 | ... | 1362 | 00 00 34.55 | +61 00 38.4 | 17.98 | 1.19 | ... |
| 1140 | 23 59 50.67 | +61 01 41.2 | 15.58 | 1.08 | ... | 1366 | 00 00 31.86 | +60 51 48.7 | 15.76 | 0.89 | ... |
| 1141 | 23 59 48.83 | +60 56 27.6 | 15.59 | 0.43 | ... | 1368 | 00 00 35.45 | +60 59 16.8 | 17.76 | 1.30 | ... |

Table 4 – continued

| Star | RA(2000) | Dec.(2000) | V | B – V | U – B | Star | RA(2000) | Dec.(2000) | V | B – V | U – B |
|------|-------------|-------------|-------|-------|-------|------|-------------|-------------|-------|-------|-------|
| 1142 | 23 59 48.58 | +60 54 55.9 | 16.75 | 0.75 | ... | 1369 | 00 00 34.42 | +60 56 17.0 | 16.72 | 1.68 | ... |
| 1143 | 23 59 48.15 | +60 52 45.3 | 17.97 | 1.15 | ... | 1370 | 00 00 34.37 | +60 55 24.0 | 16.60 | 1.62 | ... |
| 1145 | 23 59 52.16 | +61 01 56.9 | 15.26 | 0.67 | ... | 1371 | 00 00 34.24 | +60 53 47.0 | 16.69 | 1.01 | ... |
| 1146 | 23 59 49.31 | +60 54 51.9 | 16.66 | 1.48 | ... | 1372 | 00 00 34.82 | +60 54 43.0 | 16.20 | 0.71 | ... |
| 1147 | 23 59 49.43 | +60 54 30.0 | 17.56 | 1.22 | ... | 1373 | 00 00 34.78 | +60 53 52.4 | 15.45 | 1.00 | ... |
| 1148 | 23 59 51.90 | +60 59 21.9 | 16.51 | 1.08 | ... | 1374 | 00 00 36.03 | +60 54 42.1 | 17.24 | 1.15 | ... |
| 1149 | 23 59 51.37 | +60 58 06.1 | 15.19 | 0.82 | ... | 1375 | 00 00 39.55 | +61 02 38.1 | 16.98 | 1.15 | ... |
| 1152 | 23 59 53.67 | +61 02 36.0 | 17.47 | 1.18 | ... | 1376 | 00 00 37.12 | +60 57 02.4 | 16.00 | 1.08 | ... |
| 1153 | 23 59 54.32 | +61 03 11.1 | 16.90 | 0.82 | ... | 1377 | 00 00 39.04 | +61 00 55.9 | 16.67 | 1.34 | ... |
| 1154 | 23 59 51.16 | +60 55 14.9 | 17.80 | 1.65 | ... | 1379 | 00 00 37.67 | +60 57 59.9 | 15.55 | 1.02 | ... |
| 1155 | 23 59 52.79 | +60 58 19.0 | 17.38 | 0.83 | ... | 1381 | 00 00 40.36 | +61 02 47.1 | 15.80 | 1.67 | ... |
| 1157 | 23 59 53.48 | +60 59 19.6 | 15.41 | 0.78 | ... | 1382 | 00 00 40.89 | +61 03 10.5 | 15.40 | 1.48 | ... |
| 1158 | 23 59 53.70 | +60 58 05.2 | 15.70 | 1.13 | ... | 1383 | 00 00 37.34 | +60 55 18.2 | 17.14 | 0.99 | ... |
| 1159 | 23 59 53.37 | +60 56 31.1 | 16.86 | 0.86 | ... | 1384 | 00 00 38.06 | +60 56 22.3 | 17.04 | 1.35 | ... |
| 1161 | 23 59 53.61 | +60 56 42.3 | 16.31 | 0.64 | ... | 1385 | 00 00 39.47 | +60 59 25.0 | 17.78 | 1.05 | ... |
| 1162 | 23 59 53.84 | +60 56 47.8 | 17.30 | 1.07 | ... | 1386 | 00 00 36.10 | +60 51 55.4 | 17.95 | 0.91 | ... |
| 1163 | 23 59 55.67 | +61 01 05.6 | 16.52 | 2.00 | ... | 1387 | 00 00 41.30 | +61 02 53.3 | 15.70 | 1.15 | ... |
| 1164 | 23 59 59.66 | +60 58 23.2 | 15.94 | 0.88 | ... | 1388 | 00 00 36.89 | +60 52 34.9 | 16.71 | 0.75 | ... |
| 1165 | 23 59 55.58 | +60 58 07.6 | 15.57 | 0.97 | ... | 1390 | 00 00 39.24 | +60 56 41.0 | 16.81 | 1.00 | ... |
| 1166 | 23 59 55.72 | +60 58 02.6 | 16.54 | 1.06 | ... | 1391 | 00 00 39.73 | +60 56 18.0 | 17.05 | 0.98 | ... |
| 1167 | 23 59 56.69 | +61 00 12.9 | 17.40 | 1.22 | ... | 1392 | 00 00 40.08 | +60 56 38.0 | 15.57 | 1.16 | ... |
| 1170 | 23 59 55.08 | +60 52 26.3 | 16.83 | 1.07 | ... | 1393 | 00 00 39.00 | +60 54 19.5 | 16.79 | 0.84 | ... |
| 1171 | 23 59 56.83 | +60 55 47.9 | 16.50 | 0.80 | ... | 1395 | 00 00 39.72 | +60 54 38.8 | 16.34 | 1.74 | ... |
| 1172 | 23 59 59.55 | +61 01 53.2 | 16.69 | 0.94 | ... | 1396 | 00 00 43.80 | +61 02 59.9 | 15.14 | 1.08 | ... |
| 1173 | 23 59 57.85 | +60 57 25.1 | 16.45 | 0.98 | ... | 1397 | 00 00 41.17 | +60 54 50.4 | 15.25 | 1.12 | ... |
| 1174 | 00 00 00.65 | +61 03 36.3 | 15.55 | 1.06 | ... | 1398 | 00 00 40.57 | +60 52 41.2 | 17.19 | 0.86 | ... |
| 1176 | 23 59 56.51 | +60 52 55.0 | 16.74 | 1.11 | ... | 1399 | 00 00 38.87 | +60 53 10.0 | 16.01 | 0.68 | ... |
| 1177 | 00 00 00.62 | +61 01 12.0 | 17.52 | 1.34 | ... | 1400 | 00 00 43.57 | +60 58 21.3 | 15.45 | 0.76 | ... |
| 1178 | 00 00 00.20 | +60 59 46.3 | 17.25 | 0.81 | ... | 1401 | 00 00 44.40 | +60 59 36.9 | 18.65 | 1.07 | ... |
| 1179 | 00 00 01.88 | +61 03 03.4 | 15.38 | 1.07 | ... | 1402 | 00 00 43.42 | +60 56 29.0 | 15.88 | 0.82 | ... |
| 1182 | 00 00 00.15 | +60 57 08.5 | 15.28 | 1.01 | ... | 1403 | 00 00 42.83 | +60 54 31.8 | 15.47 | 0.79 | ... |
| 1183 | 00 00 00.26 | +60 56 27.9 | 16.88 | 1.92 | ... | 1404 | 00 00 43.86 | +60 54 44.7 | 16.30 | 0.96 | ... |
| 1184 | 23 59 58.63 | +60 52 34.4 | 16.41 | 1.09 | ... | 1405 | 00 00 43.28 | +60 52 49.2 | 17.41 | 0.98 | ... |
| 1185 | 00 00 00.30 | +60 56 02.3 | 16.11 | 0.75 | ... | 1406 | 00 00 44.94 | +60 55 57.8 | 16.27 | 1.06 | ... |
| 1188 | 23 59 58.82 | +60 52 06.4 | 15.79 | 0.73 | ... | 1407 | 00 00 44.69 | +60 55 09.5 | 16.07 | 1.10 | ... |
| 1189 | 23 59 59.50 | +60 53 34.1 | 17.02 | 1.53 | ... | 1409 | 00 00 44.26 | +60 52 52.3 | 15.79 | 0.76 | ... |
| 1190 | 00 00 03.53 | +61 02 52.6 | 16.24 | 1.22 | ... | 1410 | 00 00 44.22 | +60 52 20.0 | 16.66 | 1.32 | ... |
| 1191 | 00 00 00.29 | +60 54 38.8 | 15.95 | 0.94 | ... | 1411 | 00 00 49.52 | +61 02 30.2 | 15.75 | 1.66 | ... |

questions about likely intrinsic colours for the stars, but there were a number of ambiguous cases where the data from the two surveys yielded disparate solutions, for example 2MASS colours implying an early spectral type and *UBV* colours implying a late spectral type. Such cases were unimportant in the final analysis, but are curious nevertheless.

Distance moduli were calculated for individual stars by adoption of zero-age main sequence (ZAMS) values of M_V (Turner 1976a, 1979), so the values systematically underestimate $V - M_V$ for unresolved binaries and evolved stars. The resulting scatter in the variable-extinction diagram of Fig. 5 therefore contains a systematic component towards small values of $V - M_V$. Within such constraints, it is possible to discern certain trends in the data, such as the lack of any significant reddening out to distances of ~ 600 pc ($V_0 - M_V = 8.9$), with a reddening of $E(B - V) \geq 0.4$ beyond that to distances of ~ 2700 pc ($V_0 - M_V = 12.16$) or more. At the Galactic location of CG Cas ($l = 116^\circ 845, b = -1^\circ 315$), a more encompassing survey by Neckel & Klare (1980) implies a similar trend, with the reddening beginning at distances of ~ 400 – 900 pc. Apparently, the main extinction for stars in the direction

of Berkeley 58 occurs near the far side of the local spiral arm feature.

But the picture is not that simple. When the derived reddenings are compared star-for-star in the field of Berkeley 58, there are no obvious trends with spatial location, and trends with distance are difficult to establish without highly accurate luminosities for the observed stars. It can be surmised that there is additional reddening occurring on the near side of the Perseus spiral arm, given the nature of the scatter in the colour excesses. Likely members of Berkeley 58 generally have reddenings of $E(B - V) \simeq 0.70$, with larger values possibly arising from circumstellar extinction, particularly for late B-type stars where rapid rotation is common (e.g. Turner 1993, 1996a). An identical feature is observed in the adjacent cluster NGC 7790 (Takala 1988). A lower envelope trend for the reddened stars in Fig. 5 implies a ratio of total-to-selective extinction for the field of $R = A_V/E(B - V) = 2.95 \pm 0.30$ from least squares and non-parametric analyses. The value is consistent with previous studies of clusters in this region of the Galaxy (Turner 1976b), as well as with a value of $R \simeq 2.95$ expected for local extinction described by a reddening slope of 0.75 (Turner 1996a). For subsequent calculations

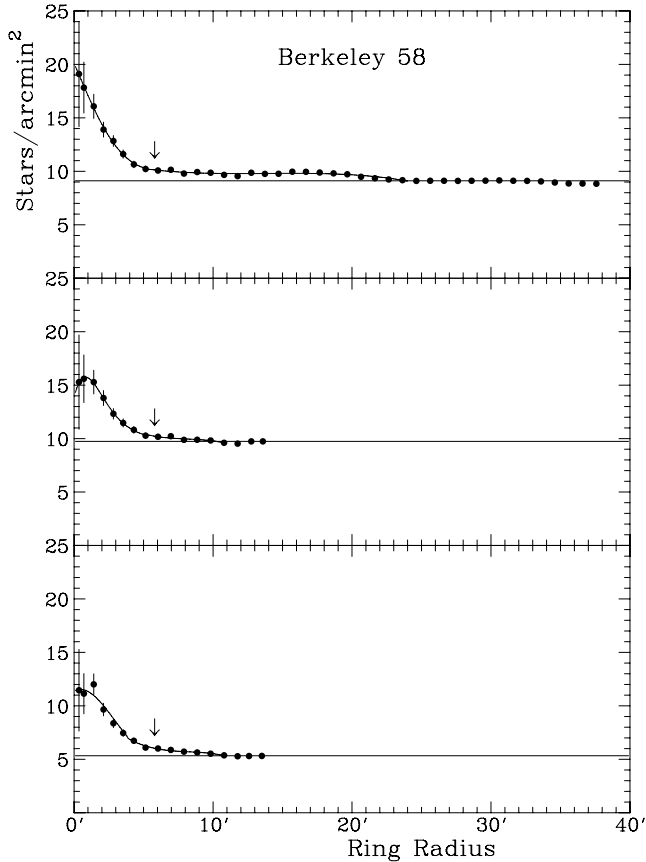


Figure 2. Star densities for the field of Berkeley 58, as measured in rings relative to the adopted cluster centre. The upper diagram contains ring counts made from the 2MASS survey, the lower two diagrams ring counts from the POSS-E plate of the field for a faint limit (middle) and a brighter limit (lower). The location of CG Cas relative to the cluster centre is indicated by an arrow.

a value of $R = 2.95$ was adopted, the exact choice affecting estimates of distance but not the derived luminosity for CG Cas as a cluster member.

An observational colour-magnitude diagram for the sampled stars is presented in Fig. 6, with a ZAMS plotted for $V - M_V = 14.29$, the apparent distance modulus at $E(B - V) = 0.70$ for points on the lower relation of Fig. 5. Such parameters provide a reasonable fit to the data, but there remain anomalies requiring further examination. For example, Fig. 6 contains reddened B-type stars more luminous than the turnoff magnitude for a cluster containing CG Cas, a point also indicated in Fig. 3, where dashed relations indicate reddening lines for B6.5 V and A2 V stars, the former corresponding to the expected turnoff colour $[(B - V)_0 = -0.13]$ for stars associated with a 4.37 d Cepheid (Turner 1996c). Clearly, the field contains a number of stars younger than the expected evolutionary age of CG Cas.

Such complications may be endemic to the field of both Berkeley 58 and NGC 7790, where the line of sight crosses the interarm region between the Sun and portions of the local spiral feature, then intercepts the Perseus spiral arm with a marked increase in space density for young B-type stars and young-to-intermediate age star clusters. The separation of spiral arm stars from cluster members is difficult but achievable, since the radial velocities for CG Cas and Berkeley 58 stars listed in Table 5 imply a conspicuous

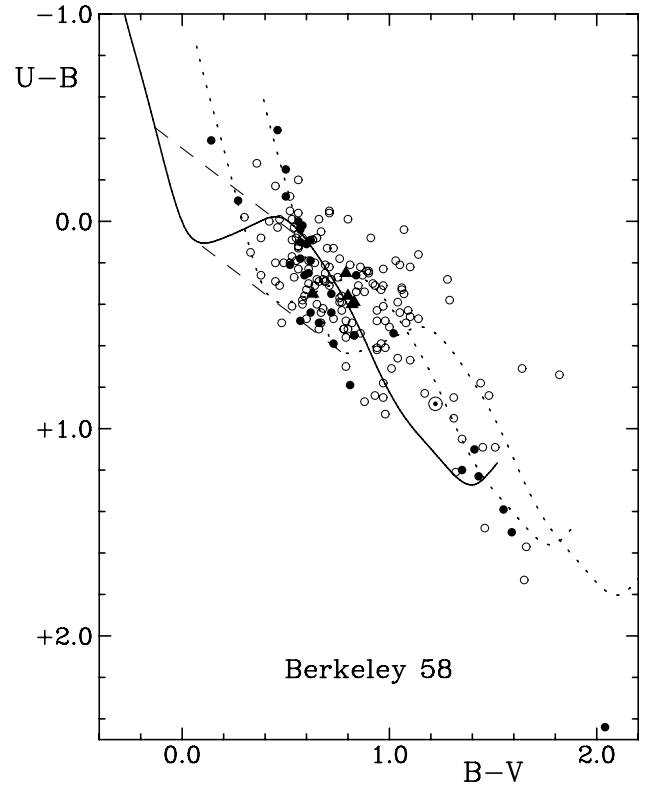


Figure 3. A UBV colour-colour diagram for observed Berkeley 58 stars: photoelectric observations (filled circles), photographic observations supplemented by CCD observations (open circles), CCD observations (filled triangles), and CG Cas (circled point). The intrinsic relation for main sequence stars is plotted as a solid line, with the same relation reddened by $E(B - V) = 0.38$ and $E(B - V) = 0.70$ shown by dotted lines. The reddening relations for stars of spectral type B6.5 V and A2 V are shown as dashed lines.

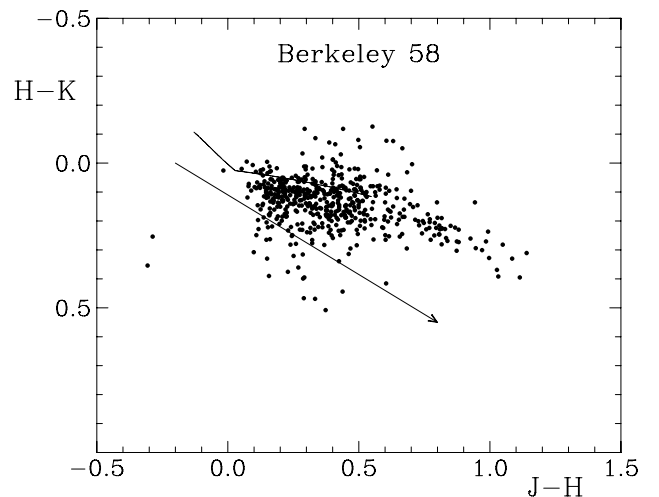


Figure 4. A 2MASS colour-colour diagram, $H - K$ versus $J - H$, for stars examined in the field of Berkeley 58, without regard to the uncertainties in the observations (Cutri et al. 2003). The intrinsic relation for main sequence stars is plotted as a solid line, as derived from the observed colours of standard stars and stars in clusters of uniform reddening. The direction of reddening in the 2MASS system is indicated.

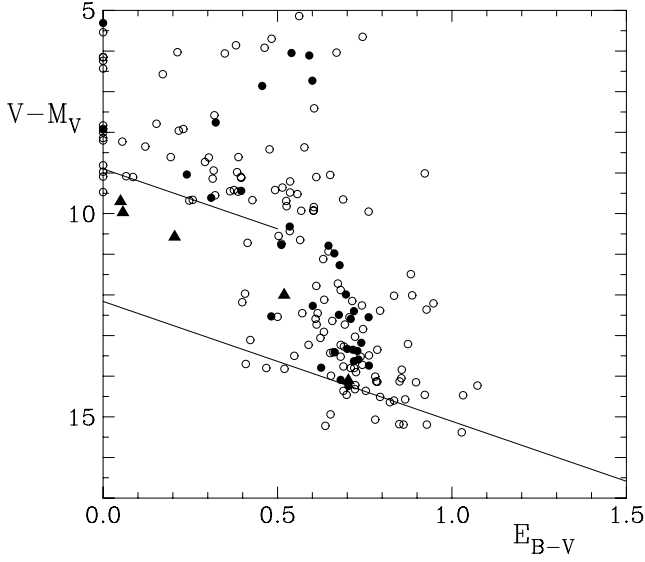


Figure 5. A variable-extinction diagram for observed Berkeley 58 stars, with symbols as in Fig. 3. Reddening relations of slope $R = A_V/E(B - V) = 2.95$ are shown corresponding to distances of $d \simeq 600$ pc ($V_0 - M_V = 8.9$) and $d \simeq 2700$ pc ($V_0 - M_V = 12.16$).

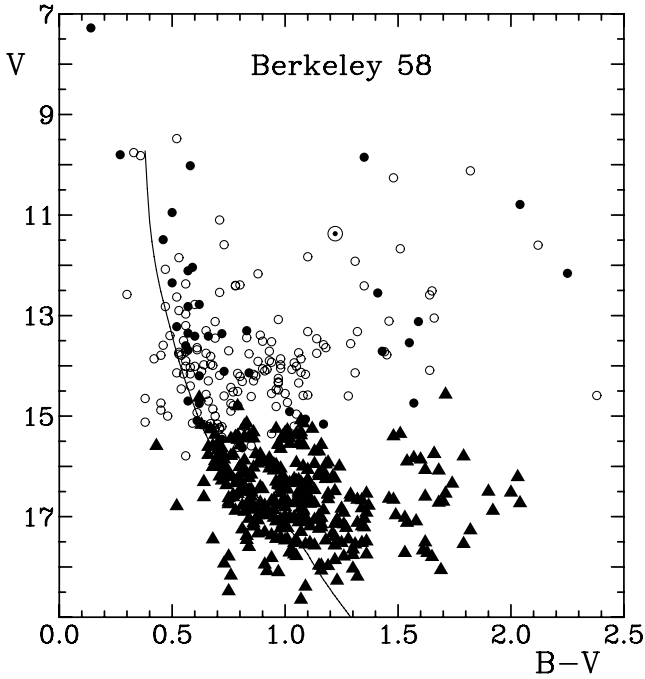


Figure 6. A colour-magnitude diagram for Berkeley 58 from all observations: photoelectric (filled circles), photographic (open circles), and CCD (triangles) data. CG Cas is the circled point. The ZAMS is depicted for $E(B - V) = 0.70$ and $V - M_V = 14.29$.

velocity difference between the cluster and spiral arm stars. The anomalously young B stars noted above are objects like stars 6 (V654 Cas), 7 and possibly 24, which have systematically more positive velocities than likely cluster members: stars 11, 18 and 23, which have radial velocities close to the systemic velocity of CG Cas (see Fig. 7, which includes radial velocity measurements from Joy 1937; Metzger et al. 1991; Gorynya et al. 1998). Except for star 11, which may be anomalous, stars with radial velocities

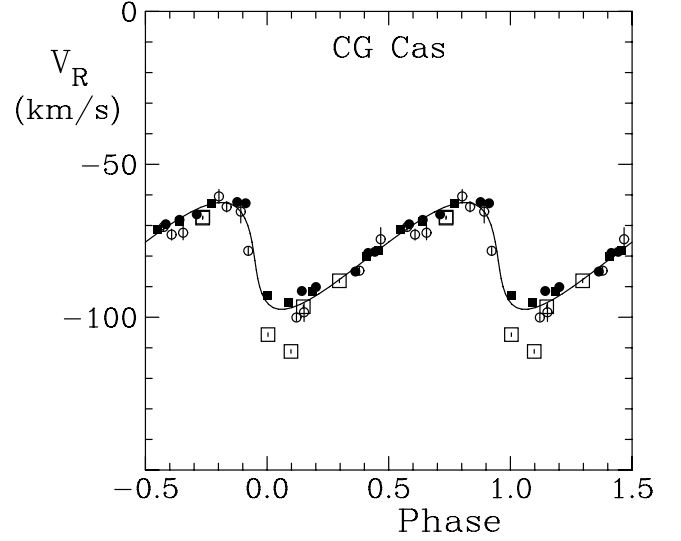


Figure 7. The radial velocity variations of CG Cas, with cited uncertainties, as measured in this paper (open circles), from Metzger et al. (1991) (filled squares) and Gorynya et al. (1998) (filled circles), and from Joy (1937) (open squares). The curve is a simple spectroscopic binary solution to the data from the first three data sets, the data from Joy (1937) exhibiting systematic deviations near velocity minimum.

Table 5. Radial velocity data for Berkeley 58 Stars.

| Star | HJD | V_R (km s ⁻¹) | Adopted V_R (km s ⁻¹) |
|----------------|--------------|--------------------------------|--|
| CG Cas | 244 5906.955 | -74.5 ± 3.8 | |
| | 244 5908.942 | -78.3 ± 1.4 | |
| | 244 5909.944 | -98.4 ± 3.0 | |
| | 244 5910.933 | -84.8 ± 1.3 | |
| | 244 5911.935 | -73.0 ± 1.8 | |
| | 244 5912.923 | -63.9 ± 1.7 | |
| | 244 6326.874 | -72.4 ± 2.3 | |
| | 244 6327.907 | -65.5 ± 3.7 | |
| | 244 6328.910 | -100.1 ± 1.2 | |
| | 244 6330.890 | -70.5 ± 1.2 | |
| 6 | 244 6331.881 | -60.5 ± 2.3 | -78.8 |
| | 24 45908.961 | -13.3 ± 3.5 | |
| | 244 6326.940 | -88.3 ± 4.1 | |
| 7 | 244 6327.955 | -47.7 ± 6.5 | -52.3 |
| | 244 5909.963 | -57.6 ± 5.3 | |
| | 244 6327.942 | -61.3 ± 2.9 | |
| 11 | 244 6331.020 | -68.7 ± 4.2 | -62.7 |
| | 244 5910.956 | -80.7 ± 1.2 | |
| | 244 6330.919 | -70.9 ± 5.1 | |
| 18 | 244 6331.909 | -70.4 ± 3.3 | -79.1 |
| | 244 5911.760 | -82.1 ± 10.1 | |
| | 244 6326.914 | -81.6 ± 3.3 | -81.6 |
| 23 | 244 6328.955 | -77.8 ± 13.0 | -77.8 |
| 24 | 244 6330.972 | -69.8 ± 8.5 | -69.8 |
| Cluster mean = | | | -79.4 ± 1.0 |

close to that of CG Cas also have spectral types near the expected B6.5 V turnoff. Unfortunately, it is not possible to identify fainter cluster members by the same technique, given the bright limit for the present radial velocity survey. Follow-up observations would be useful in that regard.

The complications arising from contamination of the cluster field by young stars in the Perseus arm and likely circumstellar reddening

for late B-type members were addressed by identifying unaffected cluster stars from their reddenings, which are close to $E(B - V)$ (B_0) = 0.70. The field of the CCD survey near the cluster centre was found to exhibit a mean reddening of $E(B - V)$ (B_0) = 0.697 ± 0.025 , that for the region of CG Cas a mean reddening of $E(B - V)$ (B_0) = 0.685 ± 0.022 . Stars with full *UBV* data were identified as likely cluster members on the basis of reddenings comparable to or larger than those values, while stars near the cluster centre lacking *U*-band data were assumed to have B_0 star colour excesses as above, but intrinsic colours adjusted for the spectral type dependence of reddening (see Fernie 1963). A-type dwarfs can suffer complications arising from the effects of rotation on their stellar continua and *UBV* colours (Turner, Usenko & Kovtyukh 2006b), so the adoption of space reddenings for such stars may circumvent potential biases introduced by dereddening their colours to the intrinsic relation for zero-age zero-rotation main-sequence stars. The resulting reddening-corrected colour–magnitude diagram for the cluster is plotted in Fig. 8 for 145 likely members, along with CG Cas and its light variations and star 5, which is considered to be a potential K giant member. The reddening for CG Cas corrected for its colour is $E(B - V) = 0.64 \pm 0.02$. A photometric reddening could be obtained from the *BVI_c* observations of Henden (1996) (see Laney & Caldwell 2007), but a field reddening was adopted as a precaution against potential bias towards large-amplitude Cepheids lying near the centre of the instability strip (unnecessary in the present case, as it turns out).

The distance to Berkeley 58 is established by 40 of its A-type ZAMS members, which yield a value of $V_0 - M_V = 12.40 \pm 0.12$

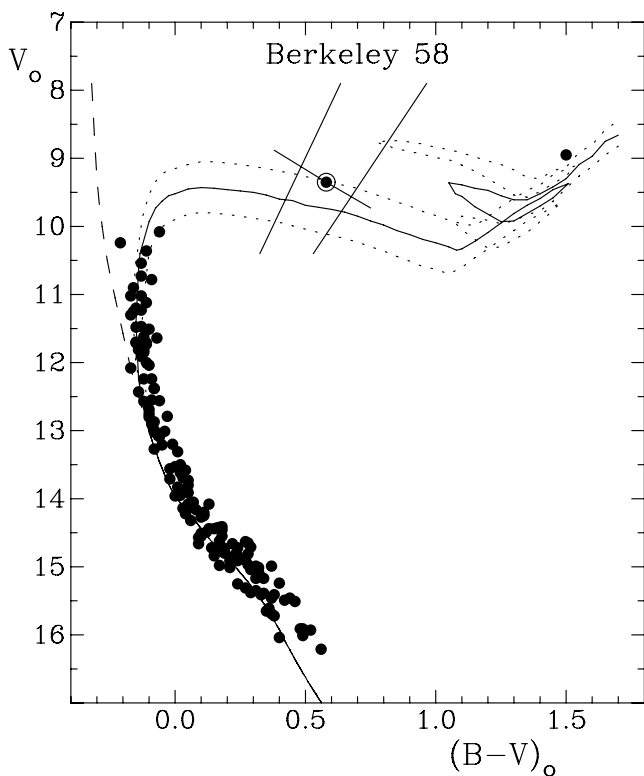


Figure 8. A reddening free colour–magnitude diagram for Berkeley 58. The dashed curve represents the ZAMS for $V_0 - M_V = 12.40$, and the solid line with dotted lines on either side represents an isochrone from Meynet et al. (1993) for $\log \tau = 8.0 \pm 0.1$. The range of light variations for CG Cas is depicted, as are the observational boundaries for the Cepheid instability strip. The red object on the evolved giant sequence is star 5.

s.d., corresponding to a distance of 3026 ± 166 pc. Except for star 11, which is conceivably a rapid rotator observed nearly pole-on, the bluest cluster stars correspond to spectral type B6 with $(B - V)_0 = -0.16$. A comparison with stellar evolutionary models (Meynet, Mermilliod & Maeder 1993) implies a cluster age of $10 \pm 1 \times 10^7$ yr ($\log \tau = 8.0 \pm 0.05$). The corresponding mass of cluster stars falling at the tip of the main-sequence red turnoff is $5.4 M_\odot$ (Meynet et al. 1993).

5 CG CASSIOPEIAE

The systemic radial velocity of CG Cas (Table 5) is a close match to the mean velocity of Berkeley 58 derived from likely cluster members 11, 18 and 23, and the evolutionary age of the cluster closely matches what is predicted for the pulsation period of the Cepheid (Turner 1996c). The luminosity of CG Cas as a likely member of Berkeley 58 is $\langle M_V \rangle = -3.06 \pm 0.12$, which matches a value of $\langle M_V \rangle = -3.04$ predicted with a Cepheid period–radius relation and the inferred effective temperature of CG Cas ($\log T_{\text{eff}} = 3.775$) from its derived intrinsic colour (Turner & Burke 2002). The case for membership of CG Cas in Berkeley 58 is very strong.

The exact evolutionary status of CG Cas can be established from the direction and rate of its period changes (Turner et al. 2006a), in conjunction with its large blue light amplitude of $\Delta B = 1.22$ (Berdnikov 2007). The period changes for CG Cas were established here from examination of archival photographic plates in the Harvard and Sternberg collections, as well as from an analysis of new and existing photometry for the star. A working ephemeris for CG Cas based upon the available data was

$$JD_{\text{max}} = 2432436.94 + 4.3656292 E,$$

where E is the number of elapsed cycles. An extensive analysis of all available observations produced the data summarized in Table 6, which lists the results for different epochs, the type of data analyzed (PG = photographic, VIS = visual telescopic observations, B = photoelectric B , and V = photoelectric V), the number of observations used to establish the times of light maximum and the source of the observations, in addition to the temporal parameters. The data are plotted in Fig. 9.

A regression analysis of the O–C data of Table 6 produced a parabolic solution for the ephemeris defined by

$$JD_{\text{max}} = 2432436.9493(\pm 0.0080) + 4.3656289(\pm 0.0000024) E + 1.1757(\pm 0.0983) \times 10^{-7} E^2,$$

which is plotted in Fig. 9. The parabolic trend corresponds to a period increase of $+0.170 \pm 0.014$ s yr^{-1} ($\log \dot{P} = -0.770 \pm 0.036$), a value typical of Cepheids lying slightly blueward of the centre of the instability strip and in the third crossing. The location of CG Cas in Fig. 8 relative to the observational boundaries of the Cepheid instability strip (Turner et al. 2006b) is consistent with that conclusion, although the stellar evolutionary models seem to require adjustments (metallicity, mixing of surface layers?) to match the observations.

6 DISCUSSION

The case for potential membership of the Cepheid CG Cas in the sparse open cluster Berkeley 58 has been studied using photometric (pe, pg, CCD) observations, spectroscopy (V_R , spectral types), star counts and O–C data for the Cepheid. The cluster Berkeley 58 is particularly difficult to separate from the young stars of the

Table 6. Times of maximum light for CG Cas.

| HJD _{max} | $\pm\sigma$ | Band | Epoch (E) | O–C (phase) | Observations (n) | Reference |
|--------------------|-------------|------|--------------|----------------|---------------------|--------------------------------|
| 241 3407.3442 | 0.0292 | PG | −4359 | +0.1714 | 55 | This paper (Harvard) |
| 241 5144.8677 | 0.0428 | PG | −3961 | +0.1746 | 7 | This paper (SAI) |
| 241 6314.8382 | 0.0338 | PG | −3693 | +0.1566 | 72 | This paper (Harvard) |
| 241 7572.0492 | 0.1070 | PG | −3405 | +0.0664 | 11 | This paper (SAI) |
| 241 9794.1315 | 0.0336 | PG | −2896 | +0.0436 | 63 | This paper (Harvard) |
| 242 3788.6688 | 0.0271 | PG | −1981 | +0.0304 | 98 | This paper (Harvard) |
| 242 6102.4299 | 0.0196 | PG | −1451 | +0.0082 | 128 | This paper (Harvard) |
| 242 6940.6916 | 0.0567 | VIS | −1259 | +0.0691 | 46 | Lange (1933) |
| 242 8023.3553 | 0.0571 | PG | −1011 | +0.0569 | 19 | This paper (SAI) |
| 242 8455.5134 | 0.0271 | PG | −912 | +0.0177 | 92 | This paper (Harvard) |
| 242 9568.7382 | 0.0559 | PG | −657 | +0.0071 | 28 | This paper (SAI) |
| 243 0847.7814 | 0.0321 | PG | −364 | −0.0790 | 81 | This paper (Harvard) |
| 243 1576.8823 | 0.0854 | PG | −197 | −0.0381 | 17 | Erleksova (1961) |
| 243 3100.4046 | 0.0430 | PG | +152 | −0.1203 | 59 | This paper (Harvard) |
| 243 3183.4566 | 0.0308 | PG | +171 | −0.0152 | 37 | This paper (SAI) |
| 243 3371.0678 | 0.0848 | PG | +214 | −0.1261 | 23 | Erleksova (1961) |
| 243 4117.6643 | 0.0350 | PG | +385 | −0.0521 | 25 | This paper (SAI) |
| 243 5174.1804 | 0.0285 | PG | +627 | −0.0182 | 74 | This paper (SAI) |
| 243 5379.4291 | 0.0443 | PG | +674 | +0.0459 | 10 | Romano (1959) |
| 243 5619.5498 | 0.1388 | PG | +729 | +0.0570 | 19 | Erleksova (1961) |
| 243 5837.7841 | 0.0168 | PG | +779 | +0.0099 | 18 | Zonn & Semeniuk (1959) |
| 243 6802.5876 | 0.0070 | B | +1000 | +0.0094 | 13 | Oosterhoff (1960) |
| 243 6802.6183 | 0.0119 | V | +1000 | +0.0401 | 15 | Oosterhoff (1960) |
| 243 6933.5492 | 0.0054 | B | +1030 | +0.0021 | 22 | Bahner, Hiltner & Kraft (1962) |
| 243 6937.9440 | 0.0085 | V | +1031 | +0.0313 | 23 | Bahner et al. (1962) |
| 243 8666.6957 | 0.0174 | PG | +1427 | −0.0061 | 41 | This paper (SAI) |
| 243 9077.0406 | 0.0299 | PG | +1521 | −0.0303 | 16 | This paper (SAI) |
| 244 0268.8346 | 0.0241 | PG | +1794 | −0.0530 | 24 | This paper (SAI) |
| 244 1146.3548 | 0.0121 | PG | +1995 | −0.0242 | 95 | This paper (SAI) |
| 244 1866.7282 | 0.0142 | PG | +2160 | +0.0204 | 55 | This paper (SAI) |
| 244 2355.6761 | 0.0178 | PG | +2272 | +0.0178 | 47 | This paper (SAI) |
| 244 2862.0722 | 0.0159 | PG | +2388 | +0.0010 | 74 | This paper (SAI) |
| 244 3045.5091 | 0.0058 | V | +2430 | +0.0815 | 71 | Chekanikhina (1982) |
| 244 3957.9197 | 0.0206 | PG | +2639 | +0.0756 | 25 | This paper (SAI) |
| 244 4844.1310 | 0.0099 | B | +2842 | +0.0643 | 9 | Berdnikov (1986) |
| 244 4852.8817 | 0.0150 | V | +2844 | +0.0837 | 11 | Berdnikov (1986) |
| 244 5189.0177 | 0.0117 | B | +2921 | +0.0663 | 8 | Berdnikov (1986) |
| 244 5189.0509 | 0.0074 | V | +2921 | +0.0995 | 8 | Berdnikov (1986) |
| 244 5394.1872 | 0.0098 | B | +2968 | +0.0512 | 14 | This paper |
| 244 5429.1355 | 0.0115 | V | +2976 | +0.0745 | 15 | This paper |
| 244 5883.1690 | 0.0061 | B | +3080 | +0.0826 | 8 | Berdnikov (1986) |
| 244 5883.1870 | 0.0086 | V | +3080 | +0.1006 | 8 | Berdnikov (1986) |
| 244 7760.4530 | 0.0042 | B | +3510 | +0.1461 | 39 | Berdnikov (1992a) |
| 244 7760.4823 | 0.0059 | V | +3510 | +0.1754 | 39 | Berdnikov (1992a) |
| 244 8118.4162 | 0.0060 | B | +3592 | +0.1277 | 18 | Berdnikov (1992b) |
| 244 8118.4546 | 0.0085 | V | +3592 | +0.1661 | 18 | Berdnikov (1992b) |
| 244 8515.7127 | 0.0043 | B | +3683 | +0.1520 | 20 | Berdnikov (1992c) |
| 244 8515.7328 | 0.0052 | V | +3683 | +0.1721 | 20 | Berdnikov (1992c) |
| 245 1458.2287 | 0.0152 | V | +4357 | +0.2341 | 27 | Wozniak et al. (2004) |

Perseus spiral arm, which raises concerns about future studies of distant open cluster calibrators for the Cepheid PL relation. Careful analysis of the available data leads to a cluster reddening of $E(B - V) (B_0) = 0.70$, a distance of 3.03 ± 0.17 kpc and an age of $10 \pm 1 \times 10^7$ yr. CG Cas is a likely member on the basis of radial velocity, location outside the cluster nucleus within the cluster coronal region, evolutionary status indicated by its period changes and light amplitude, and implied luminosity. It becomes an important Cepheid calibrator lying near the centre of the instability strip.

It may seem unusual that many potential Cepheid calibrators lie in cluster coronae rather than in cluster nuclear regions (Turner

1985), but a possible explanation relates to two dynamical lines of evidence. First, massive cluster members lie preferentially in outer regions of clusters (Burki 1978), possibly because of how protocluster interstellar clouds fragment into protostars. Secondly, as indicated by colour–magnitude diagrams for NGC 654 (Stone 1980) and other young clusters (Turner 1996b), cluster nuclear regions tend to be dominated by rapidly rotating stars, possibly the result of merged binary systems, and other close binaries, in which case potential Cepheid progenitors are less likely to evolve to the dimensions typical of pulsating variables because of restrictions on their dimensions engendered by potential physical companions. The

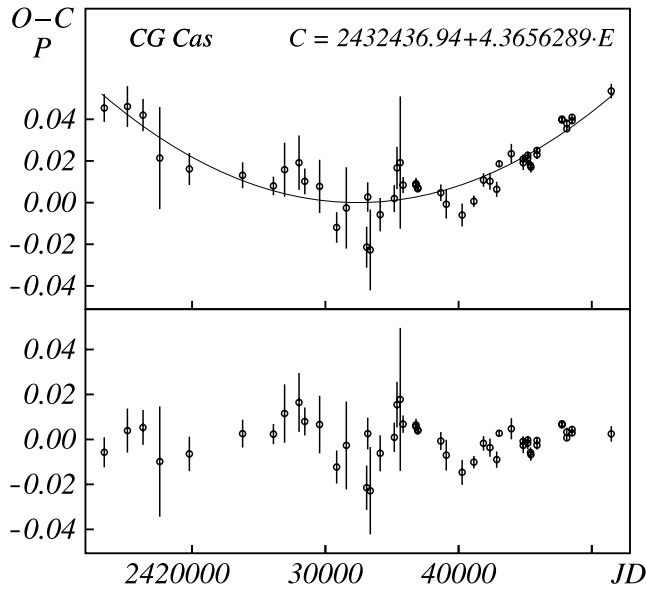


Figure 9. The differences between observed (O) and computed (C) times of light maximum for CG Cas, computed in units of pulsation phase. The upper diagram shows the actual O–C variations with their uncertainties, the lower diagram the residuals from the calculated parabolic evolutionary trend.

case of CG Cas in Berkeley 58 appears to be yet another example of the effect.

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SUPPLEMENTARY MATERIAL

The following supplementary material is available for this article:

Table 3. Photographic *UBV* data for stars in Berkeley 58.

Table 4. CCD *UBV* data for stars in the nucleus of Berkeley 58.

This material is available as part of the online article from: <http://www.blackwell-synergy.com/doi/abs/10.1111/j.1365-2966.2007.13413.x>
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